University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
Instructor: Dr. Marco A. MONTES DE OCA
Fall 2012

Homework 10

Due date: November 19, 2012

Problems

Based on Sections 15.3 through 15.9 of the book *Calculus: Early Transcendentals* 7th edition by J. Stewart.

1. The average value of a function of two variables on a plane region D is defined as $f_{\text{avg}} = \dfrac{1}{A(D)} \iint_D f(x,y)\,\mathrm{d}A$, where $A(D)$ is the area of D. Using this information, calculate the average value of $f(x,y) = x \sin y$ over the region on the plane enclosed by the curves $y = 0$, $y = x^2$, and $x = 1$.

2. In evaluating a double integral over a region D, a sum of iterated integrals was obtained as follows:
$$\iint_D f(x,y)\,\mathrm{d}A = \int_0^1 \int_0^{2y} f(x,y)\,\mathrm{d}x\mathrm{d}y + \int_1^3 \int_0^{3-y} f(x,y)\,\mathrm{d}x\mathrm{d}y.$$
Sketch the region D and express the double integral as an iterated integral with reversed order of integration.

3. Use polar coordinates to find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$.

4. Use polar coordinates to combine the sum
$$\int_{1/\sqrt{2}}^1 \int_{\sqrt{1-x^2}}^x xy\,\mathrm{d}y\mathrm{d}x + \int_1^{\sqrt{2}} \int_0^x xy\,\mathrm{d}y\mathrm{d}x + \int_{\sqrt{2}}^2 \int_0^{\sqrt{4-x^2}} xy\,\mathrm{d}y\mathrm{d}x$$
into one double integral. Then evaluate the double integral.

5. Find the area of the part of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

6. Evaluate the iterated integral $\displaystyle\int_0^1 \int_x^{2x} \int_0^y 2xyz\,\mathrm{d}z\mathrm{d}y\mathrm{d}x$. [Answer: 5/8]

7. Evaluate the iterated integral $\int_0^{\sqrt{\pi}} \int_0^x \int_0^{xz} x^2 \sin y \, \mathrm{d}y \mathrm{d}z \mathrm{d}x$. [Answer: $\pi^2/4 - 1$]

8. A new auditorium is built with a foundation in the shape of one-fourth of a circle of radius 50 feet. So, it forms a region R bounded by the graph of $x^2 + y^2 = (50)^2$ with $x \geq 0$ and $y \geq 0$. The following equations are models for the floor and ceiling. Floor: $z = \dfrac{x+y}{5}$, Ceiling: $z = 20 + \dfrac{xy}{100}$. a) Calculate the volume of the room, which is needed to determine the heating and cooling requirements. b) Find the surface area of the ceiling. [Answer: a) 30,415.74 ft^3, b) 2081.53 ft^2]

9. Solve for a in the triple integral

$$\int_0^1 \int_0^{3-a-y^2} \int_a^{4-x-y^2} \mathrm{d}z \, \mathrm{d}x \, \mathrm{d}y = \frac{14}{15}.$$

[Answer: $a = 2, 16/3$]

10. Evaluate $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} \, \mathrm{d}x \, \mathrm{d}y$. Hint: Use polar coordinates and take the limit as $r \to \infty$. [Answer: π]