University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
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Fall 2012

Homework 11

Due date: November 26, 2012

Problems

Based on Sections 15.7, 15.8, 16.1 and 16.2 of the book *Calculus: Early Transcendentals* 7th edition by J. Stewart.

1. The average value of a function of three variables over a solid region E is defined as $f_{\text{avg}} = \dfrac{1}{V(E)} \iiint_E f(x,y,z)\,\mathrm{d}V$, where $V(E)$ is the volume of E. Using this information, calculate the average value of $f(x,y,z) = x^2z + y^2z$ over the region enclosed by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$. [Answer: 1/12]

2. Evaluate the integral by changing to cylindrical coordinates: $\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{2} xz\,\mathrm{d}z\mathrm{d}x\mathrm{d}y$. [Answer: 0]

3. Use cylindrical coordinates to evaluate $\displaystyle\iiint_E (x+y+z)\,dV$, where E is the solid in the first octant that lies under the paraboloid $z = 4 - x^2 - y^2$. [Answer: $8\pi/3 + 128/15$]

4. Evaluate the integral by changing to spherical coordinates: $\displaystyle\int_0^1\int_0^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\,\mathrm{d}z\mathrm{d}y\mathrm{d}x$. [Answer: $(4\sqrt{2}-5)/15$]

5. Evaluate the integral by changing to spherical coordinates: $\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{2-\sqrt{4-x^2-y^2}}^{2+\sqrt{4-x^2-y^2}} (x^2+y^2+z^2)^{3/2}\,\mathrm{d}z\mathrm{d}y\mathrm{d}x$. [Answer: $4096\pi/21$]

6. Show that $\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sqrt{x^2+y^2+z^2}\,e^{-(x^2+y^2+z^2)}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = 2\pi$. (The improper triple integral is defined over a solid sphere as the radius of the sphere increases indefinitely.)

7. Find the gradient vector field of $f(x, y) = \sqrt{x^2 + y^2}$ and plot it using the CalcPlot3d applet. If you find a way to plot the contour level curves of this function together with the gradient vector field you will receive 5 extra points in this homework.

8. Evaluate the line integral $\displaystyle\int_C x \sin y \, ds$, where C is the line segment from $(0, 3)$ to $(4, 6)$. [Answer: $\frac{20}{9}(\sin 6 - 3\cos 6 - \sin 3)$]

9. Evaluate the line integral $\displaystyle\int_C xyz \, ds$, where C is given by $\vec{r}(t) = \langle 2\sin t, t, -2\cos t \rangle$, $0 \leq t \leq \pi$ [Answer: $\sqrt{5}\pi$]

10. The base of a circular fence with radius 10 m is given by $x = 10\cos t$, $y = 10\sin t$. The height of the fence at position (x, y) is given by the function $h(x, y) = 4 + 0.01(x^2 - y^2)$, so the height varies from 3 m to 5 m. Suppose that 1 L of paint covers 100 m^2. Determine how much paint you will need if you paint both sides of the fence. Hint: Find the lateral area under $h(x, y)$ and over the curve that describes the fence. [Answer: approximately 5.03 L of paint.]