University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
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Fall 2012

Homework 12

Due date: December 3, 2012

Problems

Based on Sections 16.1–16.3 of the book *Calculus: Early Transcendentals* 7th edition by J. Stewart. **Try first to evaluate the line integrals from scratch, then see if it is possible to use the FTC for line integrals and if it is, use it.**

1. Evaluate the line integral $\int_C \sin x\,dx + \cos y\,dy$, where C consists of the top half of the circle $x^2 + y^2 = 1$ from $(1, 0)$ to $(-1, 0)$ and the line segment from $(-1, 0)$ to $(-2, 3)$. [Answer: $\cos 1 - \cos 2 + \sin 3$]

2. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y) = \langle x + y, y - z, z^2\rangle$, and C is given by the vector function $\vec{r}(t) = \langle t^2, t^3, t^2\rangle$, for $0 \le t \le 1$. [Answer: $\frac{17}{5}$]

3. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y) = \langle z, y, -x\rangle$, and C is given by the vector function $\vec{r}(t) = \langle t, \sin t, \cos t\rangle$, for $0 \le t \le \pi$. [Answer: π]

4. Find the work done by the force field $\vec{F}(x, y) = \langle x \sin y, y\rangle$ on a particle that moves along the parabola $y = x^2$ from $(-1, 1)$ to $(2, 4)$. [Answer: approximately 8.096]

5. Determine whether or not $\vec{F} = \langle e^x \cos y, e^x \sin y\rangle$ is a conservative vector field. If it is, find a function f such that $\vec{F} = \nabla f$. [Answer: it is not.]

6. Determine whether or not $\vec{F} = \langle \frac{y^2}{1+x^2}, 2y \arctan x\rangle$ is a conservative vector field. If it is, find a function f such that $\vec{F} = \nabla f$. [Answer: it is, $f(x, y) = y^2 \arctan x + K$]

7. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, if $\vec{F} = \langle 2xz + y^2, 2xy, x^2 + 3z^2\rangle$ along the curve C: $x = t^2$, $y = t + 1$, $z = 2t - 1$, $0 \le t1$. [Answer: 7]

8. Show that the following line integral is path independent (i.e. that $\vec{F}$ is conservative) and then evaluate it. $\displaystyle\int_C \tan y\,dx + x\sec^2 y\,dy$, C is any path from $(1,0)$ to $(2,\pi/4)$.

9. Let $\vec{F} = \nabla f$, where $f(x,y) = \sin(x-2y)$. Find curves C_1 and C_2 that are not closed and satisfy a) $\int_{C_1} \vec{F}\cdot d\vec{r} = 0$, and b) $\int_{C_2} \vec{F}\cdot d\vec{r} = 1$. [Answer: there are multiple correct answers. You just have to be sure that your curves indeed satisfy the conditions.]

10. Find the curl of $\vec{F} = \langle \ln x, \ln xy, \ln xyz \rangle$ [Answer: $\left\langle \frac{1}{y}, -\frac{1}{x}, \frac{1}{x} \right\rangle$]