University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
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Fall 2012

Homework 4

Due date: October 1, 2012

Problems

Taken or adapted from Sections 13.1–13.2 the book *Calculus: Early Transcendentals* 7th edition by J. Stewart.

1. Show that the space curve described by $\vec{r}(t) = \langle 3-t, 1-t^2, 1+\frac{1}{t^2}\rangle$ passes through the points $(2,0,2)$ and $(4,0,2)$, but not through the point $(3,1,2)$.

2. If two objects travel through space along two different curves, it is often important to know whether they will collide. The curves might intersect, but we need to know whether the objects are in the same position *at the same time*. Suppose the trajectories of two particles are given by the vector functions

$$\vec{r}_1(t) = \langle t, t^2, t^3\rangle \qquad \vec{r}_2(s) = \langle 1+2s, 1+6s, 1+14s\rangle$$

   Do the particles collide? Do their paths intersect?

3. Find the unit tangent vector, $\vec{T}(t)$, to the space curve $\vec{r}(t) = \langle 2\cos(t), 2\sin(t), \tan(t)\rangle$ at $t = \pi/4$.

4. If $\vec{r}(t) = \langle e^{2t}, e^{-2t}, te^{2t}\rangle$, find $\vec{T}(0)$, $\vec{r}''(0)$, and $\vec{r}'(t)\times\vec{r}''(t)$.

5. Find parametric equations for the tangent line to the curve given by $\vec{r}(t) = \langle t, e^{-t}, 2t-t^2\rangle$, at the point $(0,1,0)$.

6. Find parametric equations for the tangent line to the curve given by $\vec{r}(t) = \langle \ln t, 2\sqrt{t}, t^2\rangle$, at the point $(0,2,1)$.

7. Evaluate $\displaystyle\int_0^1 \left(\frac{4}{1+t^2}\hat{j} + \frac{2t}{1+t^2}\hat{k}\right)\,dt$.

8. Find $\vec{r}(t)$ if $\vec{r}'(t) = \langle t, e^t, te^t\rangle$, and $\vec{r}(0) = \langle 1,1,1\rangle$.

9. Find an expression for $\frac{d}{dt}\left[\vec{u}(t) \cdot (\vec{v}(t) \times \vec{w}(t))\right]$.

10. The position of a particle is given by $\vec{r}(t) = \langle t^2, 5t, t^2 - 16t\rangle$. When is the particle moving at its minimum speed? (To remember: speed is the magnitude of the velocity).