University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
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Fall 2012

Homework 6

Due date: October 15, 2012

Problems

Based on Sections 13.4, 14.1, and 14.2 of the book *Calculus: Early Transcendentals* 7th edition by J. Stewart.

1. Show that the curvature of a plane curve in the xy-plane (that is, the graph of a function $y = f(x)$) at an inflection point is equal to zero.
2. Show that the curvature is greatest at the endpoints of the major axis, and is least at the endpoints of the minor axis, for the ellipse given by $x^2 + 4y^2 = 4$.
3. Imagine that the Batmobile is moving on the path $y = \frac{1}{3}x^3$ (x and y are measured in miles). Assume that the normal component of the Batmobile's acceleration remains constant as it moves. If the Batmobile can safely go 30 miles per hour at $(1, \frac{1}{3})$, how fast can it go at $(\frac{3}{2}, \frac{9}{8})$?
4. A highway has an exit ramp that begins at the origin of a coordinate system and follows the curve $y = \frac{1}{32}x^{5/2}$ to the point (4,1). Then it follows a circular path whose curvature is that given by the curvature at (4,1). What is the radius of the circular arc? Explain why the curve and the circular arc should have the same curvature at (4,1).
5. Evaluate the multivariable function $f(x, y) = \int_x^y (2t - 3)\, dt$ at $(0, 4)$, $(1, 4)$, $(3/2, 4)$, and $(0, 3/2)$.
6. With the help of a graphing utility, plot the surface $z = f(x, y) = \int_x^y (\frac{1}{t} - 3\sin(t))\, dt$ in the range $x \in [0.01, 10]$, and $y \in [0.01, 10]$.
7. A propane tank is constructed by welding hemispheres to the ends of a right circular cylinder. Write the volume V of the tank as a function of r and l, where r is the radius of the cylinder and hemispheres, and l is the length of the cylinder.

8. Find the limit $\lim\limits_{(x,y)\to(0,0)} \dfrac{x+y}{x^2+y}$.

9. Find the limit $\lim\limits_{(x,y)\to(1,1)} \dfrac{xy-1}{xy+1}$.

10. Find the limit $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xy+yz+xz}{x^2+y^2+z^2}$.