Name: Marco A. Montes de Oca | MATH 243 - Quiz 2
Section: 50 | September 19, 2012

Please SHOW ALL YOUR WORK as partial credit may be given; note all relevant equations, ideas, theorems, sketches, etc., to show what you know. Simplify wherever possible to make your answer more compact and neat. (Otherwise, if your answer cannot be simplified then leave it as is.) DO NOT leave your answer as a complex fraction. Answers without justification will be heavily penalized.

1. (25 pts) What is the value of α that makes the vectors $\langle 3, 2, \alpha\rangle$ and $\langle 2\alpha, 4, \alpha\rangle$ orthogonal?

   Solution:

   If $\langle 3, 2, \alpha\rangle$ and $\langle 2\alpha, 4, \alpha\rangle$ are orthogonal, then $\langle 3, 2, \alpha\rangle \cdot \langle 2\alpha, 4, \alpha\rangle = 0$. Then, $6\alpha + 8 + \alpha^2 = (\alpha + 4)(\alpha + 2) = 0$. This means that there are two values for α that would make the vectors orthogonal. One is $\alpha = -2$, and the other is $\alpha = -4$.

2. (25 pts) Find the volume of the parallelepiped determined by the vectors $\langle 2, -4, 4\rangle$, $\langle 4, -1, -2\rangle$, and $\langle 2, 3, -6\rangle$.

   Solution:

   Let $\vec{a} = \langle 2, -4, 4\rangle$, $\vec{b} = \langle 4, -1, -2\rangle$, and $\vec{c} = \langle 2, 3, -6\rangle$. The volume of the parallelepiped is given by $v = \vec{a} \cdot (\vec{b} \times \vec{c})$.

   Now,

   $$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -1 & -2 \\ 2 & 3 & -6 \end{vmatrix} = 12\hat{\imath} + 20\hat{\jmath} + 14\hat{k} = \langle 12, 20, 14\rangle.$$

   and

   $v = \vec{a} \cdot (\vec{b} \times \vec{c}) = \langle 2, -4, 4\rangle \cdot \langle 12, 20, 14\rangle = 24 - 80 + 56 = 0.$

3. (25 pts) Simplify $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$.

   Solution:

   $(\vec{a}-\vec{b}) \times (\vec{a}+\vec{b}) = \vec{a} \times (\vec{a}+\vec{b}) - \vec{b} \times (\vec{a}+\vec{b}) = \vec{a}\times\vec{a} + \vec{a}\times\vec{b} - \vec{b}\times\vec{a} - \vec{b}\times\vec{b} = \vec{a}\times\vec{b} - (-\vec{a}\times\vec{b}) = 2(\vec{a}\times\vec{b}).$

4. (25 pts) Find an equation of the plane through the point $(-1, 6, -5)$ and parallel to the plane $x + y + z + 2 = 0$.

Solution:

If the two planes are parallel, their normal vectors should be parallel. Now, the normal vector of the given plane is $\vec{n} = \langle 1, 1, 1 \rangle$. Thus, the vector equation of the plane we are looking for is:

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$, where $\vec{r} = \langle x, y, z \rangle$ and $\vec{a} = \langle -1, 6, -5 \rangle$. Expanding, we get:

$\langle x + 1, y - 6, z + 5 \rangle \cdot \langle 1, 1, 1 \rangle = (x + 1) + (y - 6) + (z + 5) = x + y + z = 0$

Bonus (10 pts): Find the direction cosines of the line joining the points $(3, 2, -4)$ and $(1, -1, 2)$.

Solution:

A vector from one point to the other is $\vec{a} = \langle 1 - 3, -1 - 2, 2 - (-4) \rangle = \langle -2, -3, 6 \rangle$. The direction cosines of this vector are the components of the unit vector $\hat{a}$, so considering that $||\vec{a}|| = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$, we have that $\hat{a} = \langle -2/7, -3/7, 6/7 \rangle$. Thus, $\cos(\alpha) = -2/7$, $\cos(\beta) = -3/7$, and $\cos(\gamma) = 6/7$.