Name: Marco A. Montes de Oca — MATH 243 - Quiz 4

Section: 50 — October 17, 2012

Please SHOW ALL YOUR WORK as partial credit may be given; note all relevant equations, ideas, theorems, sketches, etc., to show what you know. Simplify wherever possible to make your answer more compact and neat. (Otherwise, if your answer cannot be simplified then leave it as is.) DO NOT leave your answer as a complex fraction. Answers without justification will be heavily penalized.

1. (25 pts) Find the first order partial derivatives of $f(x,y) = \frac{x-y}{x+2y}$.

   Solution:

$$f_x = \frac{(x+2y)(1)-(x-y)(1)}{(x+2y)^2} = \frac{3y}{(x+2y)^2}$$
$$f_y = \frac{(x+2y)(-1)-(x-y)(2)}{(x+2y)^2} = \frac{-3x}{(x+2y)^2}$$

2. (25 pts) A particle's trajectory in space is described by the vector function $\vec{r}(t) = \langle t, 1-t^2, 1\rangle$. Its acceleration can be represented as $\vec{a}(t) = a_T\vec{T}(t) + a_N\vec{N}(t)$. Find the scalars a_T and a_N.

   Solution:

   Considering that $\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{||\vec{r}\,'(t)||}$, $a_T = \vec{a}(t)\cdot\vec{T}(t) = \frac{\vec{r}\,''(t)\cdot\vec{r}\,'(t)}{||\vec{r}\,'(t)||}$.

   So, given that $\vec{r}\,'(t) = \langle 1, -2t, 0\rangle$ and $\vec{a}(t) = \vec{r}\,''(t) = \langle 0,-2,0\rangle$, and $||\vec{r}\,'(t)|| = \sqrt{1+4t^2}$, $a_T = \frac{4t}{\sqrt{1+4t^2}}$.

   Using the Pythagorean theorem, $a_N = \sqrt{||\vec{a}(t)||^2 - a_T^2} = \sqrt{4 - \frac{16t^2}{1+4t^2}} = \sqrt{\frac{4+16t^2-16t^2}{1+4t^2}} = \frac{2}{\sqrt{1+4t^2}}$.

3. (25 pts) Find the unit tangent and the principal unit normal vectors for the curve $\vec{r}(t) = \langle 4\cos t, 4\sin t, 3t\rangle$ at $t = \pi/2$.

   Solution: The unit tangent vector is defined as $\vec{T}(t) = \frac{\vec{r}\,'(t)}{||\vec{r}\,'(t)||}$ and the principal unit normal vector as $\vec{N}(t) = \frac{\vec{T}\,'(t)}{||\vec{T}\,'(t)||}$.

$\vec{r}'(t) = \langle -4\sin t, 4\cos t, 3\rangle$ and $||\vec{r}'(t)|| = \sqrt{16\sin^2 t + 16\cos^2 t + 9} = \sqrt{16+9} = \sqrt{25} = 5$. Therefore,

$\vec{T}(t) = \frac{1}{5}\langle -4\sin t, 4\cos t, 3\rangle$. Then, $\vec{T}(\pi/2) = \langle -\frac{4}{5}, 0, \frac{3}{5}\rangle$.

Now, $\vec{T}'(t) = \frac{1}{5}\langle -4\cos t, -4\sin t, 0\rangle$ and $||\vec{T}'(t)|| = \frac{1}{5}\sqrt{16\cos^2 t + 16\sin^2 t} = \frac{1}{5}\sqrt{16} = \frac{4}{5}$. Therefore,

$\vec{N}(t) = \frac{1}{5}\frac{5}{4}\langle -4\cos t, -4\sin t, 0\rangle = \frac{1}{4}\langle -4\cos t, -4\sin t, 0\rangle = \langle -\cos t, -\sin t, 0\rangle$. Then, $\vec{N}(\pi/2) = \langle 0, -1, 0\rangle$.

4. (25 pts) Find the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy\cos y}{3x^2+y^2}$.

Solution: Approaching (0,0) from the x- or y-axis, results in the limit being 0. If we approach the origin via the line $x = y$, we obtain:

$$\lim_{(x,x)\to(0,0)} \frac{x^2\cos x}{3x^2+x^2} = \lim_{(x,x)\to(0,0)} \frac{\cos x}{4} = \frac{1}{4}.$$

Since this limit and the ones obtained previously are different, we can conclude that the limit does not exist.

Bonus (10 pts): You are hired as a consultant and your first task is to help a university increase its matriculation. You ask information about the relationship between the number of applicants, N, tuition fees, t, and room and board charges, b, and they tell you that according to their analysis $\frac{\partial N}{\partial b} < 0$ and $\frac{\partial N}{\partial t} > 0$. Would you believe this information? Explain.

Solution: No, I would not believe this information. The reason is that $\frac{\partial N}{\partial t} > 0$ means that as the tuition fee increases, so does the number of applicants, which is clearly absurd. The other parttial derivative is believable because $\frac{\partial N}{\partial b} < 0$ means that as the board and room charges increases, the number of applicants decreases.