University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
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Fall 2012

Homework 11

Due date: November 20, 2012

Problems

Based on Sections 15.3 through 15.9 of the book *Calculus: Early Transcendentals* 7th edition by J. Stewart.

1. Find the volume of the solid bounded by the planes $z = x$, $y = x$, $x + y = 2$. and $z = 0$.
2. Sketch the solid whose volume is given by the iterated integral $\int_0^1 \int_0^{1-x^2} (1 - x)\, dy\, dx$. Evaluate the integral.
3. In evaluating a double integral over a region D, a sum of iterated integrals was obtained as follows:
   $$\iint_D f(x, y)\, dA = \int_0^1 \int_0^{2y} f(x, y)\, dx\, dy + \int_1^3 \int_0^{3-y} f(x, y)\, dx\, dy.$$
   Sketch the region of integration and express the double integral as a single iterated integral with reversed order of integration.
4. Sketch the region whose area is given by the integral $\int_0^{\pi/2} \int_0^{4\cos\theta} r\, \mathrm{d}r\, \mathrm{d}\theta$ and evaluate the integral.
5. Use polar coordinates to find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$.
6. Use polar coordinates to combine the sum
   $$\int_{1/\sqrt{2}}^1 \int_{\sqrt{1-x^2}}^x xy\, \mathrm{d}y\, \mathrm{d}x + \int_1^{\sqrt{2}} \int_0^x xy\, \mathrm{d}y\, \mathrm{d}x + \int_{\sqrt{2}}^2 \int_0^{\sqrt{4-x^2}} xy\, \mathrm{d}y\, \mathrm{d}x$$
   into one double integral. Then evaluate the double integral.

7. Find the surface area of the part of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^4 = 4$. [Answer: $\pi/6(17\sqrt{17} - 5\sqrt{5})$]

8. Evaluate $\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,\mathrm{d}x\,\mathrm{d}y$. Hint: Use polar coordinates and take the limit as $r \to \infty$. [Answer: π]

9. Evaluate the integral $\displaystyle\int_0^1\int_0^1 e^{\max\{x^2,y^2\}}\,\mathrm{d}y\,\mathrm{d}x$, where $\max\{x^2, y^2\}$ means the larger of the numbers x^2 and y^2. [Answer: $e - 1$]

10. Evaluate the iterated integral $\displaystyle\int_0^1\int_x^{2x}\int_0^y 2xyz\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$. Hint: Use the same procedure you use to evaluate a double integral. [Answer: 5/8]