University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
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Fall 2012

Homework 12

Due date: November 27, 2012

Problems

Based on Sections 15.3 through 15.9 of the book *Calculus: Early Transcendentals* 7th edition by J. Stewart.

1. Evaluate the iterated integral $\int_0^1 \int_x^{2x} \int_0^y 2xyz \, \mathrm{d}z \, \mathrm{d}y \, \mathrm{d}x$. [Answer: 5/8]

2. Evaluate the iterated integral $\int_0^{\sqrt{\pi}} \int_0^x \int_0^{xz} x^2 \sin y \, \mathrm{d}y \, \mathrm{d}z \, \mathrm{d}x$. [Answer: $\pi^2/4 - 1$]

3. Sketch the solid whose volume is given by $\int_0^2 \int_0^{2-y} \int_0^{4-y^2} \mathrm{d}x \, \mathrm{d}z \, \mathrm{d}y$

4. Use a triple integral to find the volume of the solid enclosed by the surface $y = x^2$ and the planes $z = 0$ and $y + z = 1$. [Answer: 8/15]

5. The average value of a function of three variables over a solid region E is defined as $f_{\text{avg}} = \frac{1}{V(E)} \iiint_E f(x, y, z) \, \mathrm{d}V$, where $V(E)$ is the volume of E. Using this information, calculate the average value of $f(x, y, z) = x^2 z + y^2 z$ over the region enclosed by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$. [Answer: 1/12]

6. Rewrite the integral $\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z) \, \mathrm{d}z \, \mathrm{d}y \, \mathrm{d}x$ in the other five orders of integration.

7. Use cylindrical coordinates to evaluate $\iiint_E \sqrt{x^2 + y^2} \, dV$, where E is the region that lies inside the cylinder $x^2 + y^2 = 16$ and between the planes $z = -5$ and $z = 4$. [Answer: 384π]

8. Evaluate $\iiint_E x^2 \, dV$, where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$ and below the cone $z^2 = 4x^2 + 4y^2$. [Answer: $2\pi/5$]

9. Find the volume of the solid that is enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 2$. [Answer: $(4\pi/3)(\sqrt{2} - 1)$]

10. Evaluate the integral $\int_{-2}^{-2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} xz \, \mathrm{d}z \, \mathrm{d}x \, \mathrm{d}y$ by changing to cylindrical coordinates. [Answer: 0]