University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
Instructor: Dr. Marco A. MONTES DE OCA
Fall 2012

Homework 6

Due date: October 9, 2012

Problems

Based on Sections 13.3 and 13.4 of the book *Calculus: Early Transcendentals* 7th edition by J. Stewart.

1. Find the curvature κ of the curve $\vec{r}(s) = \langle 1 + 2s, 1 - 2s \rangle$, where s is the arc length parameter.

2. Find the curvature κ of the curve $\vec{r}(t) = \langle 2\cos \pi t, \sin \pi t \rangle$.

3. Find the curvature κ of the curve $\vec{r}(t) = \langle t, t^2, \frac{t^2}{2} \rangle$.

4. Find an equation of a parabola that has curvature $\frac{1}{2}$ at the origin.

5. Find the curvature of the curve with parametric equations $x = \sinh t$, $y = \cosh t$, and $z = t$ at the point (0,1,0).

6. Show that the curvature of a plane curve (i.e., a 2D curve) at an inflection point is equal to zero.

7. A car in a circular traffic exchange is traveling at twice the posted speed. By what factor is the centripetal force increased over that which would occur at the posted speed?

8. Find the speed necessary for a satellite to maintain a circular orbit 600 miles above the surface.

9. A highway has an exit ramp that begins at the origin of a coordinate system and follows the curve $y = \frac{1}{32}x^{5/2}$ to the point (4,1). Then it follows a circular path whose curvature is that given by the curvature at (4,1). What is the radius of the circular arc? Explain why the curve and the circular arc should have the same curvature at (4,1).

10. Using the fact that $\vec{r}'(t) = ||\vec{r}'||\vec{T}(t)$ and that $\vec{r}''(t) = \frac{d^2 s}{dt^2}\vec{T}(t) + \kappa \left(\frac{ds}{dt} \right)^2 \vec{N}(t)$, show that $\kappa = \frac{||\vec{r}'(t) \times \vec{r}''(t)||}{||\vec{r}'(t)||^3}$.