University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C

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Fall 2012

Homework 9

Due date: October 30, 2012

Problems

Based on Sections 14.6 of the book *Calculus: Early Transcendentals* 7th edition by J. Stewart.

1. Find the directional derivative of $f(x, y) = e^x \cos y$ at $(0, 0)$ in a direction $\pi/4$ radians with respect to the positive x-axis.
2. Find the directional derivative of $f(x, y) = 2\sqrt{x} - y^2$ at $(1, 5)$ in the direction toward the point $(4, 1)$.
3. Find the gradient of $f(x, y) = y^2/x$ at $(1, 2)$. Use it to find the rate of change of f in the direction of the vector $\frac{1}{3}\langle 2, \sqrt{5}\rangle$.
4. Find the gradient of $f(x, y, z) = y^2 e^{xyz}$ at $(0, 1, -1)$. Use it to find the rate of change of f in the direction of the vector $\langle \frac{3}{13}, \frac{4}{13}, \frac{12}{13}\rangle$.
5. Find the maximum rate of change of $f(x, y, z) = (x + y)/z$ at $(1, 1, -1)$ and the direction in which it occurs.
6. Find the points at which the direction of fastest change of the function $f(x, y) = x^2 + y^2 - 2x - 4y$ is $\langle 1, 1\rangle$.
7. Near a buoy, the depth of a lake at the point with coordinates $(x.y)$ is $z = 200 + 0.02x^2 - 0.001y^3$, where x, y and z are measured in meters. A fisherman in a small boat starts at the point (80,60) and moves toward the buoy, which is located at (0,0). Is the water under the boat getting deeper or shallower when he departs?
8. The temperature at the point (x, y, z) in space is given by $T(x, y, z) = xyz^3$. T is measured in degrees Celsius (°C); x, y, and z are measured in kilometers (km). At a certain instant of time, a spaceship is at the point $(2, 3, 1)$ and headed toward the point $(3, 4, 3)$ at a rate of 5 kilometers per second (5 km/s). At what rate is the temperature experienced by the spaceship increasing at that instant? Your answer should be in terms of degrees Celsius per second (°C/s).

9. Show that the operation of taking the gradient of a function has the following property (assume that u and v are differentiable functions of x and y and that a and b are constants): a) $\nabla(au + bv) = a\nabla u + b\nabla v$, b) $\nabla(uv) = u\nabla v + v\nabla u$, c) $\nabla(\frac{u}{v}) = \frac{v\nabla u - u\nabla v}{v^2}$, and d) $\nabla u^n = nu^{n-1}\nabla u$.

10. Are there any points on the hyperboloid $x^2 - y^2 - z^2 = 1$ where the tangent plane is parallel to the plane $z = x + y$?