Homework #8
Math 243 — Section 51
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1. That $\frac{\partial N}{\partial b} < 0$ means that as the board and room charges increase, the number of applicants decreases. This is a reasonable conclusion because applicants are usually deterred by higher costs.

That $\frac{\partial N}{\partial t} > 0$ means that as the tuition fee increases, the number of applicants increases. This phenomenon is not likely to happen, so all in all $\frac{\partial N}{\partial b} < 0$ & $\frac{\partial N}{\partial t} > 0$ is not believable.

2. $\frac{\partial z}{\partial x} = e^{x^2-y^2}(2x)$; $\frac{\partial z}{\partial y} = e^{x^2-y^2}(-2y)$

$\left.\frac{\partial z}{\partial x}\right|_{(1,-1)} = e^{1-1}(2(1)) = 2$; $\left.\frac{\partial z}{\partial y}\right|_{(1,-1)} = e^{1-1}(-2(-1)) = 2$

$\therefore$ The tangent plane at $(1,-1,1)$ is

$\underline{z-1 = 2(x-1)+2(y+1)}$ or $\underline{z = 2x+2y+1}$

3. First, we need to verify if $\vec{r}(t)$ and $\vec{p}(u)$ intersect at $(2,1,3)$.

$$\left.\begin{aligned} 2+3t &= 2 \\ 1-t^2 &= 1 \\ 3-4t+t^2 &= 3 \end{aligned}\right\} \Rightarrow t=0$$

$$\left.\begin{aligned} 1+u^2 &= 2 \\ 2u^3-1 &= 1 \\ 2u+1 &= 3 \end{aligned}\right\} \Rightarrow u=1$$

Now that we know that these curves intersect at $(2,1,3)$ we can find tangent vectors at that point:

$\vec{r}\,'(t) = \langle 3, -2t, -4+2t \rangle$
$\vec{r}\,'(0) = \langle 3, 0, -4 \rangle$

$\vec{p}\,'(u) = \langle 2u, 6u^2, 2 \rangle$
$\vec{p}\,'(1) = \langle 2, 6, 2 \rangle$

The normal vector of the tangent plane can be equal to $\vec{r}(0) \times \vec{p}(1)$, so

$$\vec{n} = \vec{r}'(0) \times \vec{p}'(1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -4 \\ 2 & 6 & 2 \end{vmatrix} = (0+24)\hat{i} - (6+8)\hat{j} + (18)\hat{k} = \langle 24, -14, 18 \rangle$$

So, an equation of the tangent plane is:

$$18(z-3) - 14(y-1) + 24(x-2) = 0$$

or

$$12x - 7y + 9z = 44$$

4. $f(x,y) = \sqrt{y + \cos^2 x}$

$$f_x(x,y) = \frac{1}{2}(y+\cos^2 x)^{-\frac{1}{2}}(2\cos x)(-\sin x)$$

$$= \frac{-\sin x \cos x}{\sqrt{y + \cos^2 x}}$$

$f_x(0,0) = 0$

$$f_y(x,y) = \frac{1}{2}(y+\cos^2 x)^{-\frac{1}{2}}(1)$$

$$= \frac{1}{2\sqrt{y+\cos^2 x}}$$

$f_y(0,0) = \frac{1}{2}$

Thus, the linearization of $f(x,y)$ @ $(0,0)$ is

$$L(x,y) = 1 + \frac{1}{2}(y-0) = \underline{1 + \frac{y}{2}}$$

5. The volume of a right circular cylinder is given by

$$V(r,h) = \pi r^2 h$$

So

$$dV = V_r(r,h)\,dr + V_h(r,h)\,dh$$
$$= 2\pi r h\,dr + \pi r^2\,dh$$

Now, $dr = 0.04\,r$ and $dh = 0.02\,h$

So

$$dV = 2(0.04)\pi r^2 h + 0.02\,\pi r^2 h$$
$$= 0.1\,\pi r^2 h$$
$$= 0.1\,V$$

$\therefore$ The maximum possible error is approximately 10% of the cylinder's volume.

6. $f(x,y) = x^4y^2 + 3x^2 - 2y$ @ $(1,1)$

$f_x(x,y) = 4x^3y^2 + 6x$

$f_y(x,y) = 2x^4y - 2$

Since:

$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

and $x_0 = 1$ & $y_0 = 1$:

$L(x,y) = 2 + 10(x-1) + 0$

$= 2 + 10x - 10 = \underline{10x - 8}$

$\underline{L(0.98, 1.05) = 1.8 \approx f(0.98, 1.05) = 1.798}$

7. $u = x^2y^3 + z^4$; $x = p + 3p^2$, $y = pe^p$, $z = p\sin p$

$$\frac{du}{dp} = \frac{\partial u}{\partial x}\frac{dx}{dp} + \frac{\partial u}{\partial y}\frac{dy}{dp} + \frac{\partial u}{\partial z}\frac{dz}{dp}$$

$$= 2xy^3(1+6p) + 3x^2y^2(e^p + pe^p) + 4z^3(\sin p + p\cos p)$$

After simplification:

$$\frac{du}{dp} = p^3\left(e^{3p}\, p(27p^3 + 81p^2 + 39p + 5) + 4\sin^4 p + 4p\sin^3 p\cos p\right)$$

8. $z = f(x,y)$, $x = g(s,t)$ and $y = h(s,t)$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$= f_x(g(s,t), h(s,t))\, g_s(s,t) + f_y(g(s,t), h(s,t))\, h_s(s,t)$$

@ $s = 1$, $t = 2$:

$$\frac{\partial z}{\partial s} = f_x(3,6)\, g_s(1,2) + f_y(3,6)\, h_s(1,2)$$

$$= 7(-1) + 8(-5) = -47$$

Similarly

$$\frac{\partial z}{\partial t} = f_x(g(s,t), h(s,t))\, g_t(s,t) + f_y(g(s,t), h(s,t))\, h_t(s,t)$$

$$= f_x(3,6)\, g_t(1,2) + f_y(3,6)\, h_t(1,2)$$

$$= 7(4) + 8(10) = 108$$

9. The volume of a right circular cone is given by

$$V(r,h) = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{2}{3}\pi r h \frac{dr}{dt} + \frac{1}{3}\pi r^2 \frac{dh}{dt}$$

$$= \frac{2}{3}\pi(120)(140)(1.8) + \frac{1}{3}\pi(120)^2(2.5)$$

$$= 20{,}160\pi + 12{,}000\pi = \underline{32{,}160\pi \ \text{in}^3/\text{s}}$$

10. Dismissed.