Name: Marco A. Montes de Oca — MATH 243 - Quiz 1

Section: 51 — September 4, 2012

Please SHOW ALL WORK as partial credit may be given; note all relevant equations, ideas, theorems, sketches, etc., to show what you know. Simplify wherever possible to make your answer more compact and neat. (Otherwise, if your answer cannot be simplified then leave it as is.) DO NOT leave your answer as a complex fraction. Answers without justification will be heavily penalized.

1. (25 pts) Find the center and radius of the sphere $x^2 + y^2 + z^2 + 2x + 8y - 4z - 28 = 0$.

   Solution:

   To find the center and radius of the sphere we first rearrange the terms:

   $x^2 + 2x + y^2 + 8y + z^2 - 4z = 28$

   We then complete squares:

   $(x^2 + 2x + 1) + (y^2 + 8y + 16) + (z^2 - 4z + 4) = 28 + 1 + 16 + 4 = 49 = 7^2$
   which can be written as:

   $(x + 1)^2 + (y + 4)^2 + (z - 2)^2 = 7^2$

   Therefore the center C is located at point $(-1, -4, 2)$ and the radius is 7.

2. (25 pts) Calculate $2\vec{a} + 3\vec{b}$ and $||\vec{a} - \vec{b}||$ if $\vec{a} = \langle 1, 1, -2\rangle$ and $\vec{b} = \langle 3, -2, 1\rangle$

   Solution:

   (i) $2\vec{a} = 2\langle 1, 1, -2\rangle = \langle 2, 2, -4\rangle$, $3\langle 3, -2, 1\rangle = \langle 9, -6, 3\rangle$, so $2\vec{a} + 3\vec{b} = \langle 11, -4, -1\rangle$.

   (ii) $\vec{a} - \vec{b} = \langle -2, 3, -3\rangle$. Therefore, $||\vec{a} - \vec{b}|| = \sqrt{(-2)^2 + 3^2 + (-3)^2} = \sqrt{4 + 9 + 9} = \sqrt{22}$.

3. (25 pts) If $\vec{r} = \langle x, y, z\rangle$ and $\vec{r_0} = \langle x_0, y_0, z_0\rangle$, what is the surface described by the equation $||\vec{r} - \vec{r_0}|| = 1$?

Solution:

The equation $||\vec{r} - \vec{r}_0|| = 1$ means that the length of the vector from the point (x_0, y_0, z_0) to some other point (x, y, z) is constant and equal to one. Therefore, $||\vec{r} - \vec{r}_0|| = 1$ represents a sphere with center (x_0, y_0, z_0) and radius one.

Another way to find the answer is to expand the equation:
$||\vec{r} - \vec{r}_0|| = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} = 1$. If we square both sides, we get $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = 1$, which is the equation of the aforementioned sphere.

4. (25 pts) If A, B, C are the vertices of a triangle, find $\vec{AB} + \vec{BC} + \vec{CA}$.

Solution:
If the vertices A, B, C form a triangle, then the head of $\vec{CA}$ must meet the tail of $\vec{AB}$. Thus, $\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$.

Bonus (10 pts): Let C be the point on the line segment between points A and B that is twice as far from B as it is from A. If we denote the origin by O, and let $\vec{a} = \vec{OA}$, $\vec{b} = \vec{OB}$, and $\vec{c} = \vec{OC}$, show that $\vec{c} = \frac{2}{3}\vec{a} + \frac{1}{3}\vec{b}$.

Solution:

We first write down the given information in vector equation form:

$\vec{b} + \vec{BA} = \vec{a}$, which means that $\vec{BA} = \vec{a} - \vec{b}$ (1). Also, $\vec{b} + \frac{2}{3}\vec{BA} = \vec{c}$ (2). Substituting (1) in (2), we get $\vec{b} + \frac{2}{3}(\vec{a} - \vec{b}) = \frac{2}{3}\vec{a} + \frac{1}{3}\vec{b} = \vec{c}$, which is what we wanted to show.