Name: Marco A. Montes de Oca | MATH 243 - Quiz 3
Section: 51 | October 2, 2012

1. (25 pts) Using appropriate differentiation rules, find $\frac{d}{dt}\left[\vec{v}(t)\cdot\vec{u}(t)\right]$, if $\vec{v}(t) = \langle \frac{1}{t}, -1, \ln t\rangle$ and $\vec{u}(t) = \langle t^2, -2t, 1\rangle$.

Solution: $\frac{d}{dt}\left[\vec{v}(t)\cdot\vec{u}(t)\right] = \vec{v}\,'(t)\cdot\vec{u}(t) + \vec{v}(t)\cdot\vec{u}\,'(t) = \left\langle -\frac{1}{t^2}, 0, \frac{1}{t}\right\rangle \cdot \langle t^2, -2t, 1\rangle + \left\langle \frac{1}{t}, -1, \ln t\right\rangle \cdot \langle 2t, -2, 0\rangle = \left(-1 + 0 + \frac{1}{t}\right) + (2 + 2 + 0) = 3 + \frac{1}{t}$

Alternatively: $\frac{d}{dt}\left[\vec{v}(t)\cdot\vec{u}(t)\right] = \frac{d}{dt}\left[t + 2t + \ln t\right] = 1 + 2 + \frac{1}{t} = 3 + \frac{1}{t}$.

2. (25 pts) Evaluate $\int_0^3 ||\vec{r}\,'(t)||\,dt$, if $\vec{r}(t) = \frac{t^2}{2}\hat{\imath} + \frac{t^3}{3}\hat{\jmath}$. What is the geometric interpretation of this integral?

Solution: $\vec{r}\,'(t) = \langle t, t^2\rangle$, so $||\vec{r}\,'(t)|| = \sqrt{t^2 + t^4}$.

$$\int_0^3 ||\vec{r}\,'(t)||\,dt = \int_0^3 \sqrt{t^2 + t^4}\,dt = \int_0^3 \sqrt{t^2(1+t^2)}\,dt = \int_0^3 t\sqrt{1+t^2}\,dt.$$

If $u = 1 + t^2$, then $du = 2t\,dt$. Therefore, $\int_0^3 t\sqrt{1+t^2}\,dt = \frac{1}{2}\int_1^{10}\sqrt{u}\,du = \frac{1}{2}\frac{u^{3/2}}{\frac{3}{2}}\Big]_1^{10} = \frac{u^{3/2}}{3}\Big]_1^{10} = \frac{10^{3/2} - 1}{3}$.

The geometric meaning of $\int_0^3 ||\vec{r}\,'(t)||\,dt$ is the arc length of the curve represented by $\vec{r}(t)$ from the point where $t = 0$ to the point where $t = 3$.

3. (25 pts) A particle moves in the xy-plane along the curve represented by the vector-valued function $\vec{r}(t) = \langle 1 - t, t^2\rangle$. Find the minimum value of the particle's speed.

Solution: $\vec{r}\,'(t) = \langle -1, 2t\rangle$, and $||\vec{r}\,'(t)|| = \sqrt{1 + 4t^2}$.

The speed will have a maximum or minimum when $\frac{d}{dt}||\vec{r}\,'(t)|| = 0$, so $\frac{d}{dt}||\vec{r}\,'(t)|| = \frac{4t}{\sqrt{1+4t^2}}$, which means that when $t = 0$, the speed will be minimum because if $t < 0$, then $\frac{d}{dt}||\vec{r}\,'(t)|| < 0$ and if $t > 0$, then $\frac{d}{dt}||\vec{r}\,'(t)|| > 0$.

The minimum value of the speed is $||\vec{r}'(0)|| = \sqrt{1+4(0)^2} = 1$.

4. (25 pts) Find the equation of a line tangent to the curve given by $\vec{r}(t) = \langle t, t^2, 0\rangle$ at (2,4,0).

Solution: We need a point on the line and the direction vector. The point is (2,4,0). The direction vector should be tangent to $\vec{r}(t)$ when $t = 2$. Such a vector is $\vec{r}'(t) = \langle 1, 2t, 0\rangle$ at $t = 2$, so the direction vector $\vec{v} = \vec{r}'(2) = \langle 1, 4, 0\rangle$.

The equation of the line is thus: $\vec{q}(s) = \langle 2, 4, 0\rangle + s\langle 1, 4, 0\rangle = \langle 2+s, 4+4s, 0\rangle$.

Bonus (10 pts): What is the length of the curve $\vec{r}(t) = \langle 1+2t, t, -2+3t\rangle$ from $t = 0$ to $t = 2$?

Solution: The length of $\vec{r}(t)$ from $t = 0$ to $t = 2$ is equal to $\int_0^2 ||\vec{r}'(t)||\,dt = \int_0^2 ||\langle 2, 1, 3\rangle||\,dt = \int_0^2 \sqrt{4+1+9}\,dt = \int_0^2 \sqrt{14}\,dt = \sqrt{14}t\Big]_0^2 = 2\sqrt{14}$.