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Section: 50

MATH 243 - Quiz 1

September 18, 2013

Please SHOW ALL YOUR WORK as partial credit may be given; note all relevant equations, ideas, theorems, sketches, etc., to show what you know. Simplify wherever possible to make your answer more compact and neat. (Otherwise, if your answer cannot be simplified then leave it as is.) DO NOT leave your answer as a complex fraction. Answers without justification will be heavily penalized.

1. (25 pts) The vector $\vec{a} = \langle 3, 5, 4 \rangle$ makes three angles, α, β, and γ, with the positive $x-$, $y-$, and $z-$ axes, respectively. Find these angles.

+5pts (x3)

$\vec{a} \cdot \hat{i} = 3 = \|\vec{a}\| \, \cancel{\|\hat{i}\|}^{1} \cos\alpha = \sqrt{9+25+16}\,\cos\alpha = \sqrt{50}\cos\alpha = 5\sqrt{2}\cos\alpha \Rightarrow \alpha = \cos^{-1}\left(\frac{3}{5\sqrt{2}}\right)$ 3 pts

Similarly,

$\beta = \cos^{-1}\left(\frac{5}{5\sqrt{2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$ 3 pts

$\gamma = \cos^{-1}\left(\frac{4}{5\sqrt{2}}\right)$ 3 pts

2. (25 pts) The points $(2, 0, 0)$, $(3, 3, 0)$, $(0, 0, 1)$, and $(-1, -3, 1)$ form a parallelogram. Calculate its area.

$$\left.\begin{aligned}\vec{A} &= \langle 3-2, 3-0, 0-0 \rangle = \langle 1, 3, 0 \rangle \\ \vec{B} &= \langle 0-2, 0-0, 1-0 \rangle = \langle -2, 0, 1 \rangle\end{aligned}\right\} \text{10 pts}$$

Area $= \|\vec{A} \times \vec{B}\|$, so

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 0 \\ -2 & 0 & 1 \end{vmatrix} = \langle 3, -1, 6 \rangle \Big\} \text{10 pts}$$

$$\|\vec{A} \times \vec{B}\| = \sqrt{9+1+36} = \boxed{\sqrt{46}} \Big\} \text{5 pts}$$

3. (25 pts) At which point does the line through $(1,0,1)$ and $(0,2,0)$ intersect the plane $2x-y+z=3$?

Direction vector $\vec{v} = \langle 0-1, 2-0, 0-1 \rangle = \langle -1, 2, -1 \rangle$ } 5 pts

$\vec{r}(t) = \langle 1,0,1 \rangle + t\langle -1,2,-1 \rangle = \langle 1-t, 2t, 1-t \rangle$ } 10 pts

In equation of plane:

$2(1-t) - (2t) + (1-t) = 3$

$2 - 2t - 2t + 1 - t = 3$

$-5t + 3 = 3$

$-5t = 0 \Rightarrow t = 0$

$\therefore$

$x = 1 - 0 = 1$

$y = 2(0) = 0$

$z = 1 - 0 = 1$

Point: $(1,0,1)$

} 10 pts

4. (25 pts) Find a linear equation of the plane through $(1,1,1)$ and perpendicular to the line whose parametric equations are $x = 1+t$, $y = 2t$, $z = 2-t$.

$\vec{v} = \langle 1, 2, -1 \rangle$ Direction vector of line.

We note that $\vec{n} = \vec{v}$, so

} 15 pts

$1(x-1) + 2(y-1) - 1(z-1) = 0$

$x - 1 + 2y - 2 - z + 1 = 0$

$x + 2y - z - 2 = 0$

$x + 2y - z = 2$

} 10 pts