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Section: 51 September 17, 2013

Please SHOW ALL YOUR WORK as partial credit may be given; note all relevant equations, ideas, theorems, sketches, etc., to show what you know. Simplify wherever possible to make your answer more compact and neat. (Otherwise, if your answer cannot be simplified then leave it as is.) DO NOT leave your answer as a complex fraction. Answers without justification will be heavily penalized.

1. (25 pts) The points $(1, 1, 0)$, $(0, 3, 0)$, $(0, 0, 2)$ form a triangle. Calculate its area.

$$\left.\begin{aligned}\vec{A} &= \langle 0-1, 3-1, 0-0\rangle = \langle -1, 2, 0\rangle \\ \vec{B} &= \langle 0-1, 0-1, 2-0\rangle = \langle -1, -1, 2\rangle\end{aligned}\right\} \text{5 pts}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -1 & -1 & 2 \end{vmatrix} = (2(2)-(-1)(0))\hat{i} - ((-1)(2)-(-1)(0))\hat{j} + ((-1)(-1)-(-1)(2))\hat{k}$$

$$= 4\hat{i} + 2\hat{j} + 3\hat{k} = \langle 4, 2, 3\rangle \Big\} \text{10 pts}$$

$$\text{Area} = \frac{1}{2}\|\vec{A}\times\vec{B}\| = \frac{1}{2}\sqrt{16+4+9} = \boxed{\frac{1}{2}\sqrt{29}} \Big\} \text{10 pts}$$

2. (25 pts) If $\vec{a} = \langle 1, -1, 3\rangle$ and $\vec{b} = \langle 4, 1, -2\rangle$, find $\tan\theta$, where $0 \leq \theta \leq \pi$ is the angle between $\vec{a}$ and $\vec{b}$.

$$\text{5pts}\Big\{ \vec{a}\cdot\vec{b} = \|\vec{a}\|\,\|\vec{b}\|\cos\theta \;;\; \|\vec{a}\times\vec{b}\| = \|\vec{a}\|\,\|\vec{b}\|\sin\theta \implies$$

$$\text{10pts}\Big\{ \tan\theta = \frac{\|\vec{a}\times\vec{b}\|}{\vec{a}\cdot\vec{b}}$$

$$\text{5pts}\Big\{ \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 4 & 1 & -2 \end{vmatrix} = ((-1)(-2)-(1)(3))\hat{i} - ((1)(-2)-(4)(3))\hat{j} + ((1)(1)-(4)(-1))\hat{k}$$

$$= -\hat{i} + 14\hat{j} + 5\hat{k} = \langle -1, 14, 5\rangle$$

$$\|\vec{a}\times\vec{b}\| = \sqrt{(-1)^2+(14)^2+5^2} = \sqrt{1+196+25} = \sqrt{222}$$

$$\text{5pts}\Big\{ \vec{a}\cdot\vec{b} = \langle 1, -1, 3\rangle\cdot\langle 4, 1, -2\rangle = (1)(4)+(-1)(1)+(3)(-2) = 4-1-6 = -3$$

$$\therefore \boxed{\tan\theta = \frac{\sqrt{222}}{-3}}$$

3. (25 pts) At which point does the line through $(1,1,0)$ and $(0,2,-1)$ intersect the plane $-2x + y + z = 5$?

$\vec{v} = \langle 0-1, 2-1, -1-0 \rangle = \langle -1, 1, -1 \rangle$ } 5pts

$\vec{r}(t) = \langle 1,1,0 \rangle + t\langle -1, 1, -1 \rangle = \langle 1-t, 1+t, -t \rangle \Rightarrow$ } 10pts

$-2(1-t) + (1+t) + (-t) = 5 \Rightarrow$

$-2 + 2t + 1 + \cancel{t} - \cancel{t} = 5$

$2t - 1 = 5$

$t = \frac{5+1}{2} = \frac{6}{2} = 3 \Rightarrow$

10pts { $x = 1-3 = -2, \quad y = 1+3 = 4, \quad z = -3$

$\therefore$ Point $(-2, 4, -3)$

4. (25 pts) Find a linear equation of the plane through $(-1,0,2)$ and perpendicular to the line whose parametric equations are $x = 1 + 2t$, $y = 1 - t$, $z = -3t$.

From the parametric equations:

$\vec{v} = \langle 2, -1, -3 \rangle$

So, $\vec{v} = \vec{n} \Rightarrow$ } 15 pts

Plane:

$2(x-(-1)) - (y-0) - 3(z-2) = 0$

$2x + 2 - y - 3z + 6 = 0$

$\underline{2x - y - 3z = -8}$ } 10 pts