University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
Instructor: Marco A. Montes de Oca
Spring 2012

Homeworks 10 & 11

Name: ______________________________ **Section:** ________

Due date: April 3, 2012 (Section 50)
April 2, 2012 (Section 51)

Problems

Taken or adapted from Chapter 14 Review of the book *MATH 241/242/243 University of Delaware* by J. Stewart. Each exercise is worth 10 points for a total of 200 points (100 points per homework).

1. Find and sketch the domain of the following functions:

   a) $f(x, y) = \sqrt{y} + \sqrt{25 - x^2 - y^2}$,
   b) $f(x, y) = \arcsin(x^2 + y^2 - 2)$.

2. Find the limit, if it exists, or show that the limit does not exist:

   a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$,
   b) $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{yz}{x^2 + 4y^2 + 9z^2}$.

3. Find the first partial derivatives of $f(x, t) = \sqrt{x}\ln(t)$.

4. Find the first partial derivatives of $w = \dfrac{e^v}{u + v^2}$.

5. Find the first partial derivatives of $f(x, y) = \displaystyle\int_y^x \cos(t^2)\,dt$.

6. Find the first partial derivatives of $u = \sin\left(\sum_{i=1}^{n} i x_i\right)$.

7. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of $yz = \ln(x+z)$.

8. Find all the second partial derivatives of $z = \arctan\left(\frac{x+y}{1-xy}\right)$.

9. The level curves of a function f are shown in Fig. 1. Determine whether the following partial derivatives are positive, negative or zero at the point P. (EXPLAIN your answer.) a) f_x, b) f_y, c) f_{xx}, d) f_{xy}, e) f_{yy}.

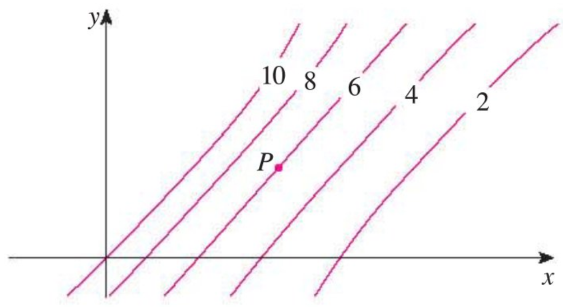

Figure 1: Problem 9

10. Verify that the function $z = \ln(e^x + e^y)$ is a solution (that is, its partial derivatives satisfy the given constraints) of the differential equations

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$$

and

$$\frac{\partial^2 z}{\partial x^2}\frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 = 0$$

11. Find the equation of the tangent plane to the surface defined by $z = y\ln(x)$ at the point $(1, 4, 0)$.

12. Find the equation of the tangent plane to the surface defined by $z = e^{x^2-y^2}$ at the point $(1, -1, 1)$.

13. Find the linear approximation of the function $f(x, y) = \ln(x-3y)$ at $(7, 2)$ and use it to approximate $f(6.9, 2.06)$.

14. Suppose you need to know an equation of the tangent plane to a surface S at the point $P(2, 1, 3)$ [**Corrected from previous version where P's coordinates were** $(1, 2, 3)$]. You don't have an equation for S but you know that the curves

$$\vec{r}_1(t) = \langle 2+3t, 1-t^2, 3-4t+t^2\rangle$$

and

$$\vec{r}_2(u) = \langle 1+u^2, 2u^3-1, 2u+1\rangle$$

both lie on S. Find an equation of the tangent plane at P.

15. Use the chain rule for partial derivatives to find dz/dt given that $z = \arctan\left(\frac{y}{x}\right)$, where $x = e^t$ and $y = 1 - e^{-t}$.

16. If $N = \dfrac{p+q}{p+r}$, where $p = u + vw$, $q = v + uw$, and $r = w + uv$. Find the value of $\frac{\partial N}{\partial v}$, when $u = 2$, $v = 3$, and $w = 4$.

17. Find the gradient of $f(x,y) = y^2/x$. Evaluate the gradient at $P(1,2)$. Find the rate of change of f at P in the direction of the vector $\vec{u} = \frac{1}{3}(2\hat{\imath} + \sqrt{5}\hat{\jmath})$.

18. Find the maximum rate of change of $f(x,y) = sin(xy)$ at $P(1,0)$ and the direction at which it occurs.

19. Find all the points at which the direction of fastest change of the function $f(x,y) = x^2 + y^2 - 2x - 4y$ is parallel to $\hat{\imath} + \hat{\jmath}$.

20. Assuming that u and v are differentiable functions of x and y and that a, b are constants, show that the operation of taking the gradient of a function has the following properties: a) $\nabla(au + bv) = a\nabla u + b\nabla v$, b) $\nabla(uv) = u\nabla v + v\nabla u$, c) $\nabla\left(\frac{u}{v}\right) = \frac{v\nabla u - u\nabla v}{v^2}$, d) $\nabla u^n = nu^{n-1}\nabla u$.