

University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C

Instructor: Marco A. Montes de Oca

Spring 2012

Homework 14

Name: _____ **Section:** _____

Due date: April 24, 2012 (Section 50)
April 23, 2012 (Section 51)

Problems

Taken or adapted from the book *MATH 241/242/243 University of Delaware* by J. Stewart. Each exercise is worth 10 points for a total of 100 points.

1. Use spherical coordinates to evaluate $\int_{-2}^{-2} \int_0^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2} dz dx dy$
2. Give five other iterated integrals that are equal to $\int_0^2 \int_0^{y^3} \int_0^{y^2} f(x, y, z) dz dx dy$
3. Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 3$. [Here you may consider evaluating this integral in cylindrical coordinates.]
4. Use the transformation $x = u^2$, $y = v^2$, and $z = w^2$ to find the volume of the region bounded by the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = 1$ and the coordinate planes.
5. Evaluate the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy dz dy dx$ by changing to spherical coordinates.
6. Show that $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{x^2 + y^2 + z^2} e^{-(x^2+y^2+z^2)} dx dy dz = 2\pi$. [Here you have to integrate over a solid sphere of radius a and then take the limit as the radius increases indefinitely.]

7. Find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.
8. A swimming pool is circular with a 40-ft diameter. The depth is constant along east-west lines and increases linearly from 2-ft at the south end to 7 ft at the north end. Find the volume of water in the pool.
9. What is the role of the absolute value of the Jacobian when we change variables in a multiple integral?
10. Give an example of a function $f(x, y)$ that violates the assumptions of Fubini's theorem.