University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
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Spring 2012

Homework 15

Name: ______________________________ **Section:** ________

Due date: May 1, 2012 (Section 50)
April 30, 2012 (Section 51)

Problems

Taken or adapted from the books *MATH 241/242/243 University of Delaware* by J. Stewart and *Vector Analysis* by Spiegel et al. Each exercise is worth 10 points for a total of 100 points.

1. A particle moves in a velocity field $\vec{V}(x, y) = \langle x^2, x + y^2 \rangle$. If it is at position $(2, 1)$ at time $t = 3$, estimate its location at time $t = 3.01$.
2. Evaluate $\int_C xyz \, \mathrm{d}s$, where C is given by the parametric equations $x = 2\sin(t)$, $y = t$, $z = -2\cos(t)$, $0 \leq t \leq \pi$.
3. Show that the field $\vec{F} = \langle 1 - ye^{-x}, e^{-x} \rangle$ is conservative and evaluate $\int_C \vec{F} \cdot \mathrm{d}\vec{r}$, where C is any path from $(0, 1)$ to $(1, 2)$.
4. Let $\vec{F} = (2y + 3)\hat{i} + (xz)\hat{j} + (yz - x)\hat{k}$. Evaluate $\int_C \vec{F} \cdot \mathrm{d}\vec{r}$ along the following path C: the straight lines from $(0, 0, 0)$ to $(0, 0, 1)$, then to $(0, 1, 1)$, and then to $(2, 1, 1)$.
5. Find the work done in moving a particle in the force field $\vec{F} = \langle 3x^2, 2xz - y, z \rangle$ along the space curve $\vec{r}(t) = \langle 2t^2, t, 4t^2 - t \rangle$, from $t = 0$ to $t = 1$.
6. A 160-lb man carries a 25-lb can of paint up a helical staircase that encircles a silo with a radius of 20 ft. If the silo is 90 ft high and the man makes exactly three complete revolutions, how much work is done by the man against gravity in climbing to the top?
7. Suppose there is a hole in the can of paint in the previous exercise and 9 lb of paint leaks steadily out of the can during the man's ascent. How much work is done?

8. Use the Fundamental Theorem of Calculus for Line Integrals to find the work done in moving an object in the field $\vec{F} = \langle y^2 \cos(x) + z^3, 2y \sin(x) - 4, 3xz^2 + 2 \rangle$ from $(0, 1, -1)$ to $(\pi/2, -1, 2)$.

9. Let $\vec{F} = \nabla f$, where $f(x, y) = \sin(x - 2y)$. Find curves C_1 and C_2, that are not closed and satisfy the equations $\int_{C_1} \vec{F} \cdot \mathrm{d}\vec{r} = 0$ and $\int_{C_2} \vec{F} \cdot \mathrm{d}\vec{r} = 1$.

10. Evaluate the line integral $\oint_C (x - y)\,\mathrm{d}x + (x + y)\,\mathrm{d}y$, where C is the circle with center the origin and radius 2.