University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
Instructor: Marco A. Montes de Oca
Spring 2012

Homework 5

Name: ______________________________ **Section:** ________

Due date: February 23, 2012 (Section 50)
February 22, 2012 (Section 51)

Problems

Taken or adapted from Section 12.4 of the book *MATH 241/242/243 University of Delaware* by J. Stewart. Each exercise is worth 10 points for a total of 100 points.

1. Exercise # 12.4–1. Find the cross product $\vec{a} \times \vec{b}$ of $\vec{a} = \langle 6, 0, -2 \rangle$ and $\vec{b} = \langle 0, 8, 0 \rangle$ and verify that it is orthogonal to both $\vec{a}$ and $\vec{b}$.

2. Exercise # 12.4–7. Find the cross product $\vec{a} \times \vec{b}$ of $\vec{a} = \langle t, t^2, t^3 \rangle$ and $\vec{b} = \langle 1, 2t, 3t^2 \rangle$ and verify that it is orthogonal to both $\vec{a}$ and $\vec{b}$.

3. Exercise # 12.4–8. If $\vec{a} = \hat{\imath} - 2\hat{k}$ and $\vec{b} = \hat{\jmath} + \hat{k}$, find $\vec{a} \times \vec{b}$. Sketch $\vec{a}$, $\vec{b}$, and $\vec{a} \times \vec{b}$ as vectors starting at the origin.

4. Exercise # 12.4–17. If $\vec{a} = \langle 1, 2, 1 \rangle$ and $\vec{b} = \langle 0, 1, 3 \rangle$, find $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$.

5. Exercise # 12.4–21. Show that $\vec{0} \times \vec{a} = \vec{0} = \vec{a} \times \vec{0}$ for any vector $\vec{a}$ in V_3.

6. Exercise # 12.4–35. Find the volume of the parallelepiped with adjacent edges PQ, PR, and PS. The points are $P(2, 0, -1)$, $Q(4, 1, 0)$, $R(3, -1, 1)$, and $S(2, -2, -2)$.

7. Exercise # 12.4–40. Find the magnitude of the torque about P if a 36-lb force is applied as shown in Figure 1.

8. Exercise # 12.4–42. Let $\vec{v} = 5\hat{\jmath}$ and let $\vec{u}$ be a vector with length 3 that starts at the origin and rotates in the xy-plane. Find the maximum and minimum values of the length of the vector $\vec{u} \times \vec{v}$. In what direction does $\vec{u} \times \vec{v}$ point?

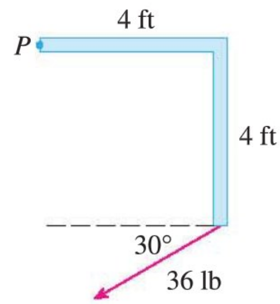

Figure 1: Torque problem

9. Exercise Review Chapter # 12–4. Calculate the given quantity if $\vec{a} = \hat{\imath} + j - 2\hat{k}$, $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$, and $\vec{c} = \hat{\jmath} - 5\hat{k}$.

   - $2\vec{a} + 3\vec{b}$
   - $|\vec{b}|$
   - $\vec{a} \cdot \vec{b}$
   - $\vec{a} \times \vec{b}$
   - $|\vec{b} \times \vec{c}|$
   - $\vec{a} \cdot (\vec{b} \times \vec{c})$
   - $\vec{c} \times \vec{c}$
   - $\vec{a} \times (\vec{b} \times \vec{c})$
   - $\text{comp}_{\vec{a}}\vec{b}$
   - $\text{proj}_{\vec{a}}\vec{b}$
   - The angle between $\vec{a}$ and $\vec{b}$

10. Exercise # 12.4–53 (*Calculus: Early Transcendentals* 7th edition by J. Stewart.) Suppose that $\vec{a} \neq \vec{0}$.

   - If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, does it follow that $\vec{b} = \vec{c}$? If it does, explain. If it does not, give a counterexample.
   - If $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, does it follow that $\vec{b} = \vec{c}$? If it does, explain. If it does not, give a counterexample.
   - If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, does it follow that $\vec{b} = \vec{c}$? If it does, explain. If it does not, give a counterexample.