University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
Instructor: Marco A. Montes de Oca
Spring 2012

Homework 8

Name: ______________________________ **Section:** ________

Due date: March 13, 2012 (Section 50)
March 12, 2012 (Section 51)

Problems

Taken or adapted from Sections 13.1, 13.2, and 13.3 of the book *MATH 241/242/243 University of Delaware* by J. Stewart. Each exercise is worth 10 points for a total of 100 points.

1. Exercise # 13.1–10. Sketch the curve with the vector equation $\vec{r}(t) = \langle 1+t, 3t, -t\rangle$. Indicate with an arrow the direction in which t increases.

2. Exercise # 13.1–28. At what points does the helix $\vec{r}(t) = \langle \sin(t), \cos(t), t\rangle$, intersect the sphere $x^2 + y^2 + z^2 = 5$?

3. Exercise # 13.1–36. Find a vector function that represents the curve of intersection of the surfaces $x^2 + y^2 = 4$ and $z = xy$.

4. Exercise # 13.2–26. Find the parametric equations for the tangent line to the curve represented by the vector function $\vec{r}(t) = \langle \ln(t), 2\sqrt{t}, t^2\rangle$ at the point $(0, 2, 1)$.

5. Exercise # 13.2–32. At what point do the curves $\vec{r}_1(t) = \langle t, 1-t, 3+t^2\rangle$ and $\vec{r}_2(s) = \langle 3-s, s-2, s^2\rangle$ intersect? Find their angle of intersection.

6. Exercise # 13.2–40. Find $\vec{r}(t)$ if $\vec{r}\,'(t) = \langle t, e^t, te^t\rangle$ and $\vec{r}(0) = \langle 1, 1, 1\rangle$.

7. Exercise # 13.3–4. Find the length of the curve $\vec{r}(t) = \cos(t)\hat{\imath} + \sin(t)\hat{\jmath} + \ln(\cos(t))\hat{k}$, $0 \le t \le \pi/4$.

8. Exercise # 13.3–20. Use $\kappa(t) = \frac{|\vec{T}\,'(t)|}{|\vec{r}\,'(t)|}$ to find the curvature of $\vec{r}(t) = \langle t, t^2/2, t^2\rangle$.

9. Exercise # 13.3–32. Find an equation of a parabola that has curvature 4 at the origin.

10. Exercise # 13.3–40. Use the fact that

$$\kappa(t) = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3}$$

to show that the curvature of a plane curve with parametric equations $x = f(t)$ and $y = g(t)$ is

$$\kappa(t) = \frac{|f'(t)g''(t) - g'(t)f''(t)|}{|[f'(t)]^2 + [g'(t)]^2|^{3/2}}$$