University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
Instructor: Marco A. Montes de Oca
Spring 2012

Homework 9

Name: _____ **Section:** _____

Due date: March 20, 2012 (Section 50)
March 19, 2012 (Section 51)

Problems

Taken or adapted from Chapter 13 Review of the book *MATH 241/242/243 University of Delaware* by J. Stewart. Each exercise is worth 10 points, except the last one, which is worth 40 points, for a total of 100 points.

1. Assuming that a function f is twice differentiable, show that the curvature of the curve $y = f(x)$ is zero at an inflection point.
2. Prove (in general) or disprove (through an example) the following statement: *Different parametrizations of the same curve result in identical tangent vectors at a given point on the curve.*
3. Find the length of the curve $\vec{r}(t) = \langle 2t^{3/2}, \cos(2t), \sin(2t)\rangle$, $0 \leq t \leq 1$.
4. Find the curvature of the ellipse $x = 3\cos(t)$, $y = 4\sin(t)$ at the points $(3, 0)$ and $(0, 4)$.
5. A particle starts at the origin with initial velocity $\hat{i} - \hat{j} + 3\hat{k}$. Its acceleration is $\vec{a} = 6t\hat{i} + 12t^2\hat{j} - 6t\hat{k}$. Find its position function.
6. Find the curvature of the curve with parametric equations
$$x = \int_0^t \sin\left(\frac{1}{2}\pi\theta^2\right) d\theta, \quad y = \int_0^t \cos\left(\frac{1}{2}\pi\theta^2\right) d\theta$$
7. [40 points] If a projectile is fired with an angle of elevation α and initial speed v, then parametric equations for its trajectory are

$$x = vt\cos(\alpha), \quad y = vt\sin(\alpha) - \frac{1}{2}gt^2$$

What value of α maximizes the total distance (that is, the arc length) traveled by the projectile? (You may use MAPLE to tackle this problem.)