University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
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Spring 2013

Homework 11

Due date: April 29, 2013

Problems

Based on Sections 16.3-16.5 of the book *Calculus: Early Transcendentals* 7th edition by J. Stewart.

1. Determine whether the vector field $\boldsymbol{F}(x,y) = \langle 2xy + y^{-2}, x^2 - 2xy^{-3} \rangle$, $y > 0$ is conservative. If it is, find f such that $\nabla f = \boldsymbol{F}$.

2. Use the FTC4LIs to evaluate $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ if $\boldsymbol{F} = \langle x^2, y^2 \rangle$, and C is the arc of the parabola $y = 2x^2$ from $(-1, 2)$ to $(2, 8)$.

3. Show that $\int_C xe^y\,dx + \left(\frac{1}{2}(x^2e^y + y^2)\right)\,dy$ is path-independent.

4. Show that $\int_C x^2e^y\,dx + \left(\frac{1}{2}(x^2e^y + xy^2)\right)\,dy + xy\,dz$ is path-dependent.

5. Show that any vector field of the form $\boldsymbol{F}(x,y,z) = \langle f(x), g(y), h(z) \rangle$, where f, g, h are differentiable functions, is irrotational (that is, that $\nabla \times \boldsymbol{F} = \boldsymbol{0}$).

6. Evaluate $\oint_C xy\,dx + x^3\,dy$ directly and using Green's Theorem. C is the rectangle with vertices $(0,0)$, $(3,0)$, $(3,1)$, and $(0,1)$.

7. Use Green's Theorem to evaluate $\oint_C xy^2\,dx + 8x^2y\,dy$, where C is the triangle with vertices $(0,0)$, $(2,2)$, and $(2,4)$.

8. Use Green's Theorem to evaluate $\oint_C y^4\,dx + 2xy^3\,dy$, where C is the positively defined (that is, it is traced in a CCW direction) ellipse $x^2 + 2y^2 = 2$.

9. Calculate $\oint_C \boldsymbol{F} \cdot d\boldsymbol{r}$, where $\boldsymbol{F} = \langle e^x + 2y, 8x - \tan y \rangle$ and C is a positively oriented boundary curve of a region D that has area 5.

10. Use Green's Theorem to evaluate $\oint_C \sqrt[5]{1 + x^7}\, dx + xy^2\, dy$, where C is the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 3)$.