University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
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Spring 2013

Homework 12

Due date: May 14, 2013

Problems

Based on Sections 16.5-16.8 of the book *Calculus: Early Transcendentals* 7th edition by J. Stewart.

1. Find the surface area of the part of the plane $2x + y + z = 5$ that lies in the first octant.
2. Find the area of the part of the surface $z = x + y^2$ inside the cylinder $x^2 + y^2 = 4$.
3. Find the area of the part of the sphere $x^2+y^2+z^2 = b^2$ that lies inside the cylinder $x^2+y^2 = a^2$ where $0 < a < b$.
4. Evaluate the surface integral $\iint_S xyz\, dS$, where S is the surface with parametric representation $x = u\cos v$, $y = u \sin v$, $z = u$, where $0 \leq u \leq 1$, $0 \leq v \leq \pi/2$.
5. Find the flux of $\boldsymbol{F}$ through S by evaluating $\iint_S \boldsymbol{F} \cdot d\boldsymbol{S}$, where $\boldsymbol{F}(x, y, z) = \langle x, 2y, 3z\rangle$ and S is the cube with vertices $(\pm1, \pm1, \pm1)$.
6. A fluid has density 1200 kg/m^3 and flows in a velocity field $\boldsymbol{v} = \langle y, x, 4\rangle$, where x,y, and z, are measured in meters and the components of $\boldsymbol{v}$ are measured in m/s. Find the rate of flow (that is, the flux) of the fluid outward through the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$.
7. Use the Divergence Theorem to calculate the surface integral $\oiint_S \boldsymbol{F} \cdot d\boldsymbol{S}$, where $\boldsymbol{F} = \langle x^2yz, xy^2z, xyz^2\rangle$, where S is the surface bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 1$.
8. Use the Divergence Theorem to calculate the surface integral $\oiint_S \boldsymbol{F} \cdot d\boldsymbol{S}$, where $\boldsymbol{F} = \langle x^3 + y^3, y^3 + z^3, z^3 + x^3\rangle$, where S is a sphere centered at the origin with radius 2.
9. Evaluate $\oiint_S (2x + 2y + z^2)dS$, where S is a sphere centered at the origin with radius 2.

10. Show that $\oiint_S \nabla \times \boldsymbol{F} \cdot d\boldsymbol{S} = 0$, assuming that the components of $\boldsymbol{F}$ have continuous second order partial derivatives and S is the boundary surface of a simple solid region.