University of Delaware
Department of Mathematical Sciences

MATH-243 – Analytical Geometry and Calculus C
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Spring 2013

Homework 6

Due date: March 18, 2013

Problems

Based on Sections 14.4–14.6 of the book *Calculus: Early Transcendentals* 7th edition by J. Stewart.

1. Find the differential of the function $z = \sin^2(xy)$.
2. Use differentials to estimate the amount of metal in a closed cylindrical can that is 4 in. high and 2 in. in diameter if the metal in the top and bottom is 0.013 in. thick and the metal in the side is 0.01 in. thick.
3. Use the Chain Rule to find $\frac{dz}{dt}$ of $z = \arctan(y/x)$, $x = e^t$, $y = 1 - e^{-t}$.
4. Use the Chain Rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ of $z = e^{x-y^2}$, where $x = ts^2$ and $y = st^2$.
5. The temperature at a point (x, y) is $T(x, y)$, measured in degrees Celsius. A bug crawls so that its position after t seconds is given by $x = \sqrt{1+t}$, $y = 2 + \frac{1}{3}t$, where x and y are measured in centimeters. If $T(x, y) = 4x + 3y + 1$, what is the rate of change of the temperature on the bug's path after 3 seconds?
6. Find the directional derivative of $f(x, y) = ye^{-x}$ at $(0, 4)$ in the direction of the vector $\vec{v} = -\hat{\imath} + \hat{\jmath}$.
7. Find the gradient of $f(x, y, z) = y^3 e^{xyz}$, evaluate it at $P(0, 1, -1)$, and find the rate of change of f at P in the direction of the vector $\vec{v} = \langle 3/13, 4/13, 12/13 \rangle$.
8. Find the maximum rate of change of $f(x, y) = \sin(xy)$ at $(1, 0)$ and the direction in which it occurs.
9. Find the directions in which the directional derivative of $f(x, y) = ye^{-xy}$ at $(0, 2)$ has the value 1.
10. Show that the ellipsoid $3x^2 + 2y^2 + z^2 = 9$ and the sphere $x^2 + y^2 + z^2 - 8x - 6y - 8z + 24 = 0$ are tangent to each other at the point $(1, 1, 2)$. (This means that they have a common tangent plane at the point.)