Math 243 - Section 51

Spring 2013

Homework #2

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1. a) scalar projection: $\text{comp}_{\vec{a}}\vec{b} = \dfrac{\vec{b}\cdot\vec{a}}{\|\vec{a}\|} =$

$$\frac{\langle 0, 1, \frac{1}{2}\rangle \cdot \langle -2, 3, -6\rangle}{\sqrt{4+9+36}} = \frac{3-3}{7} = 0$$

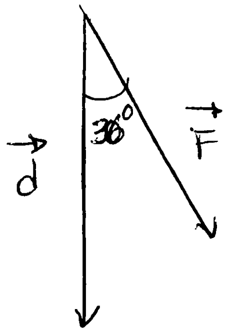
vector projection: $\text{proj}_{\vec{a}}\vec{b} = 0\,\hat{a} = \vec{0}$

b) $\text{comp}_{\vec{a}}\vec{b} = \dfrac{\langle 1, -1, 1\rangle \cdot \langle 1, 1, 1\rangle}{\sqrt{1+1+1}} = \dfrac{1}{\sqrt{3}}$

$$\text{proj}_{\vec{a}}\vec{b} = \frac{1}{\sqrt{3}}\,\hat{a} = \frac{1}{\sqrt{3}}\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\rangle$$

$$= \left\langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right\rangle$$

2.



$$\vec{W} = \vec{F} \cdot \vec{d} = \|\vec{F}\| \, \|\vec{d}\| \cos\theta$$
$$= (400)(120) \cos 36^\circ$$
$$= 38832.81 \text{ ft-lb.}$$

3.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & -1 \\ -1 & 2 & 3 \end{vmatrix} = [(-2)(3) - (2)(-1)]\,\hat{i} - [(2)(3) - (-1)(-1)]\,\hat{j} + [(2)(2) - (-1)(-2)]\,\hat{k} =$$
$$-4\hat{i} - 5\hat{j} + 2\hat{k}$$

$$\vec{b} \times \vec{a} = -\vec{a} \times \vec{b} = 4\hat{i} + 5\hat{j} - 2\hat{k}$$

4. a) $\vec{n} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 2 \\ -1 & -1 & 3 \end{vmatrix} = \underline{2\hat{i} + 4\hat{j} + 2\hat{k}}$

$$\text{Area}_{PQR} = \frac{1}{2} \|PQ \times \vec{PR}\| = \frac{1}{2}\sqrt{4+16+4} =$$
$$= \frac{1}{2}\sqrt{24} = \underline{\sqrt{6}}$$

b) $\vec{n} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & -2 \\ 5 & 5 & 1 \end{vmatrix} = \underline{13\hat{i} - 14\hat{j} + 5\hat{k}}$

$\text{Area}_{PQR} = \frac{1}{2} \| \vec{PQ} \times \vec{PR} \| = \frac{1}{2} \sqrt{(13)^2 + (-14)^2 + 5^2} =$

$\underline{\frac{1}{2} \sqrt{390}}$

5. If $\vec{a} = \langle 1, 2, 1 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$

and $\vec{a} \times \vec{v} = \langle 3, 1, -5 \rangle \Rightarrow$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ v_1 & v_2 & v_3 \end{vmatrix} = (2v_3 - v_2)\hat{i} - (v_3 - v_1)\hat{j} + (v_2 - 2v_1)\hat{k}$$
$$= \langle 2v_3 - v_2, v_1 - v_3, v_2 - 2v_1 \rangle = \langle 3, 1, -5 \rangle$$

$\Rightarrow$ $2v_3 - v_2 = 3$

$v_1 - v_3 = 1 \Rightarrow v_1 = 1 + v_3$

$v_2 - 2v_1 = -5 \Rightarrow v_2 = -5 + 2v_1$

So, if $v_1 = \alpha$, then

$\underline{\vec{v} = \langle \alpha, 2\alpha - 5, \alpha - 1 \rangle}$

b) If there were a vector $\vec{v}$ such that

$\langle 1,2,1\rangle \times \vec{v} = \langle 3,1,5\rangle$, then

$\langle 1,2,1\rangle \cdot \langle 3,1,5\rangle = 0$

However,

$\langle 1,2,1\rangle \cdot \langle 3,1,5\rangle = 3+2+5 = 10 \neq 0$

$\therefore$ $\vec{v}$ does not exist.

6. If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are coplanar, then

$\vec{u} \cdot (\vec{v} \times \vec{w}) = 0$.

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 5 & 9 & -4 \end{vmatrix} = (4-0)\hat{i} - (-12-0)\hat{j} + (27+5)\hat{k}$$

$$= \langle 4, 12, 32\rangle$$

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \langle 1,5,-2\rangle \cdot \langle 4,12,32\rangle$$

$$= 4 + 60 - 64 = \underline{0}$$

7. Let $\vec{v} = \langle 0-(-7), 1-2, 3-4 \rangle$

$= \langle 7, -1, -1 \rangle$

$\Rightarrow$ The line is represented by

$\vec{r} = \langle -7, 2, 4 \rangle + t \langle 7, -1, -1 \rangle$

$= \langle -7+7t, 2-t, 4-t \rangle$

So the parametric equations are

$$\left.\begin{aligned} x &= -7+7t \\ y &= 2-t \\ z &= 4-t \end{aligned}\right\}$$

The symmetric equations are

$$\frac{x+7}{7} = 2-y = 4-z$$

8. L_1: $\langle 1, 1, 0 \rangle + t\langle 1, -1, 2 \rangle = \langle 1+t, 1-t, 2t \rangle$

L_2: $\langle 2, 0, 2 \rangle + s\langle -1, 1, 0 \rangle = \langle 2-s, -s, 2 \rangle$

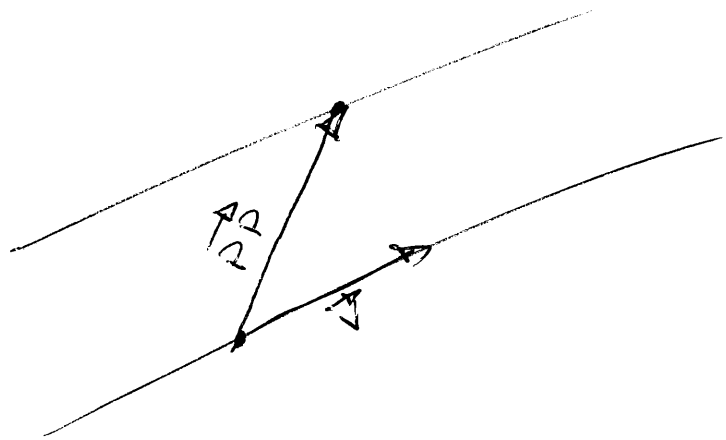
$\Rightarrow$ $1+t = 2-s$
$1-t = -s$
$2t = 2 \Rightarrow t = 1 \Rightarrow 1-1 = -s = 0 \Rightarrow s = 0$

So, the intersection point is $\underline{(2,0,2)}$.

9. We derived the expression in class.

$$d = \frac{\|\vec{PP} \times \vec{v}\|}{\|\vec{v}\|}$$

where



10. Two lines are parallel if their direction vectors are parallel.

L_1: $\vec{v}_1 = \langle -1, 2, 1 \rangle$

L_2: $\vec{v}_2 = \langle 3, -6, -3 \rangle$

$\vec{v}_1$ is parallel to $\vec{v}_2$ because

$$-3\vec{v}_1 = \vec{v}_2$$

A point on L1 is $(2, 3, 4)$ $(t=0)$
and on L2 is $(0, 1, 4)$ $(t=0)$ $\Rightarrow$

$$\vec{PP} = \langle 0-2, 1-3, 4-4 \rangle = \langle -2, -2, 0 \rangle$$

So

$$\vec{PP} \times \vec{v}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -2 & 0 \\ -1 & 2 & 1 \end{vmatrix} = (-2-0)\hat{i} - (-2-0)\hat{j} + (-4-2)\hat{k}$$

$$= \langle -2, 2, -6 \rangle$$

$$\|\vec{PP} \times \vec{v}_1\| = \sqrt{4+4+36} = \sqrt{44} = 2\sqrt{11}$$

$$\|\vec{v}_1\| = \sqrt{1+4+1} = \sqrt{6}$$

$$\Rightarrow \quad \boxed{d = \frac{\|\vec{PP} \times \vec{v}_1\|}{\|\vec{v}_1\|} = \frac{2\sqrt{11}}{\sqrt{6}}}$$