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Section: 51 February 13, 2013

Please SHOW ALL YOUR WORK as partial credit may be given; note all relevant equations, ideas, theorems, sketches, etc., to show what you know. Simplify wherever possible to make your answer more compact and neat. (Otherwise, if your answer cannot be simplified then leave it as is.) DO NOT leave your answer as a complex fraction. Answers without justification will be heavily penalized.

1. (25 pts) Use vectors to determine whether the points $A(1, 2, 4)$, $B(2, 5, 0)$, and $C(0, 1, 5)$ lie on a straight line (that is, determine whether the points are collinear).

   Solution: We can define vectors $\vec{AB}$ and $\vec{AC}$. If the points A, B and C are collinear, then the vectors $\vec{AB}$ and $\vec{AC}$ should be parallel, that is, there should a scalar α such that $\vec{AB} = \alpha\vec{AC}$. If no such number exists, then the points are not collinear.

   Thus, $\vec{AB} = \langle 1, 3, -4\rangle$ and $\vec{AC} = \langle -1, -1, 1\rangle$. If $\vec{AB} = \alpha\vec{AC}$, then $\langle 1, 3, -4\rangle = \alpha\langle -1, -1, 1\rangle = \langle -\alpha, -\alpha, \alpha\rangle$. It follows that $-\alpha = 1$, $-\alpha = 3$, and $\alpha = -4$. Since there is no value for α that could satisfy these equations simultaneously, we can conclude that the points A, B and C are not collinear.

2. (25 pts) If $\vec{u} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$, what should be the value of α if we need that $||\alpha\vec{u}|| = 6$?

   Solution: Since $||\alpha\vec{u}|| = |\alpha|||\vec{u}||$, then $|\alpha|(\sqrt{4 + 4 + 1}) = |\alpha|(3) = 6$, or $|\alpha| = 2$. Thus, $\alpha = \pm 2$.

3. (25 pts) What can be said about the vectors $\vec{u}$ and $\vec{v}$ if (a) the projection of $\vec{u}$ onto $\vec{v}$ equals $\vec{u}$ and (b) the projection of $\vec{u}$ onto $\vec{v}$ equals $\vec{0}$?

   Solution: a) $\vec{u}$ and $\vec{v}$ are parallel. (b) $\vec{u}$ and $\vec{v}$ are orthogonal.

4. (25 pts) Find the area of the parallelogram with vertices $A(1, 1, 1)$, $B(2, 3, 4)$, $C(6, 5, 2)$, $D(7, 7, 5)$.

   Solution: We notice that $\vec{AB} = \langle 1, 2, 3\rangle = \vec{CD}$ and $\vec{AC} = \langle 5, 4, 1\rangle = \vec{BD}$

   Thus, the area of the parallelogram is equal to $||\vec{AB}\times\vec{AC}|| = ||\langle -10, 14, -6\rangle|| = \sqrt{100 + 196 + 36} = \sqrt{332}$.

Bonus (10 pts): Consider the vectors $\vec{u} = \langle \cos\alpha, \sin\alpha, 0\rangle$ and $\vec{v} = \langle \cos\beta, \sin\beta, 0\rangle$, where $\alpha > \beta$. Find the dot product of the vectors and use the result to prove the identity $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

Solution: $\vec{u} \cdot \vec{v} = \cos\alpha\cos\beta + \sin\alpha\sin\beta = (\cancelto{1}{\sqrt{\cos^2\alpha + \sin^2\alpha}})(\cancelto{1}{\sqrt{\cos^2\beta + \sin^2\beta}})\cos(\alpha - \beta) = \cos(\alpha - \beta)$.