Name: Marco A. Montes de Oca — MATH 243 - Quiz 2

Section: 51 — March 6, 2013

Please SHOW ALL YOUR WORK as partial credit may be given; note all relevant equations, ideas, theorems, sketches, etc., to show what you know. Simplify wherever possible to make your answer more compact and neat. (Otherwise, if your answer cannot be simplified then leave it as is.) DO NOT leave your answer as a complex fraction. Answers without justification will be heavily penalized.

1. (25 pts) Find the velocity, acceleration, and speed of a particle with the position function $\vec{r}(t) = e^t\langle \cos t, \sin t, t\rangle$.

   Solution:

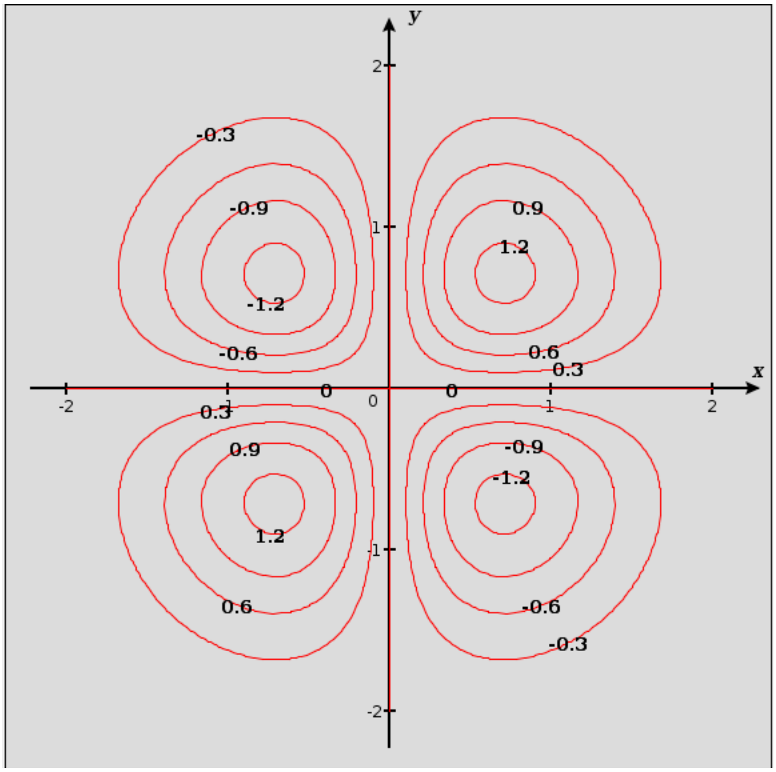
   Since $\vec{r}(t) = f(t)\vec{u}(t)$, then $\vec{v}(t) = \vec{r}\,'(t) = f'(t)\vec{u}(t)+f(t)\vec{u}\,'(t) = e^t\langle \cos t, \sin t, t\rangle + e^t\langle -\sin t, \cos t, 1\rangle = e^t\langle \cos t - \sin t, \sin t + \cos t, 1+t\rangle$.

   Using the same idea, $\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t) = e^t\langle \cos t - \sin t, \sin t + \cos t, 1+t\rangle + e^t\langle -\sin t - \cos t, \cos t - \sin t, 1\rangle = e^t\langle -2\sin t, 2\cos t, 2+t\rangle$

   Finally, the speed of the particle is $||\vec{v}(t)|| = ||e^t\langle \cos t - \sin t, \sin t + \cos t, 1+t\rangle|| = e^t\sqrt{2+(1+t)^2}$

2. (25 pts) A contour map of a function is shown in the next page. Use it to estimate the value of the function at the points $(1.5, 1)$ and $(-1, 1)$.

   Solution: a) $0.3 < f(1.5, 1) < 0.6$,
   b) $-1.2 < f(-1, 1) < -0.9$



3. (25 pts) Find the limit, if it exists, or show that the limit does not exist: $\displaystyle\lim_{(x,y)\to(2,1)} \frac{4-xy}{x^2+3y^2}$.

   Solution: The limit can be found by direct substitution. Therefore, $\displaystyle\lim_{(x,y)\to(2,1)} \frac{4-xy}{x^2+3y^2} = \frac{2}{7}$.

4. (25 pts) Find all the second partial derivatives of $f(x,y) = xe^{-2y}$.

   Solution:

   $f_x(x,y) = e^{-2y}$
   $f_y(x,y) = xe^{-2y}(-2) = -2xe^{-2y}$.

   $f_{xx}(x,y) = 0$
   $f_{yy}(x,y) = -2xe^{-2y}(-2) = 4xe^{-2y}$
   $f_{xy}(x,y) = e^{-2y}(-2) = -2e^{-2y}$
   $f_{yx}(x,y) = -2e^{-2y}$

Bonus (10 pts): Using the contour plot of Problem 2, estimate the sign of $\frac{\partial f}{\partial y}$ at $(1, 0.2)$.

Solution: Since the function at $(1, 0.2)$ increases in the positive y direction, then $\frac{\partial f}{\partial y} > 0$.