Name: Marco A. Montes de Oca MATH 243 - Quiz 4
Section: 51 April 10, 2013

Please, SHOW ALL YOUR WORK as partial credit may be given; note all relevant equations, ideas, theorems, sketches, etc., to show what you know. Simplify wherever possible to make your answer more compact and neat. (Otherwise, if your answer cannot be simplified then leave it as is.) DO NOT leave your answer as a complex fraction. Answers without justification will be heavily penalized.

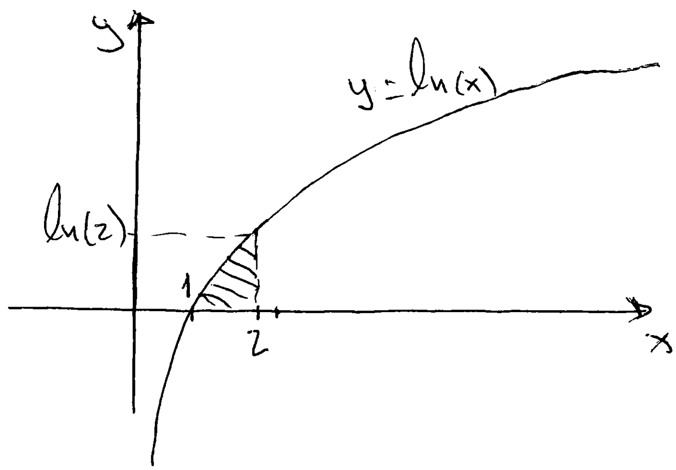
1. (25 pts) Calculate the iterated integral $\int_1^3 \int_1^5 \frac{\ln y}{xy}\,dy\,dx$.

Solution: $\int_1^3 \int_1^5 \frac{\ln y}{xy}\,dy\,dx = \int_1^3 \left[\frac{(\ln y)^2}{2x}\right]_1^5 dx = \frac{(\ln 5)^2}{2}\int_1^3 \frac{1}{x}\,dx = \frac{(\ln 5)^2}{2}\left[\ln x\right]_1^3 = \frac{1}{2}(\ln 5)^2 \ln 3.$

2. (25 pts) Evaluate the iterated integral $\int_0^2 \int_y^{2y} xy\,dx\,dy$.

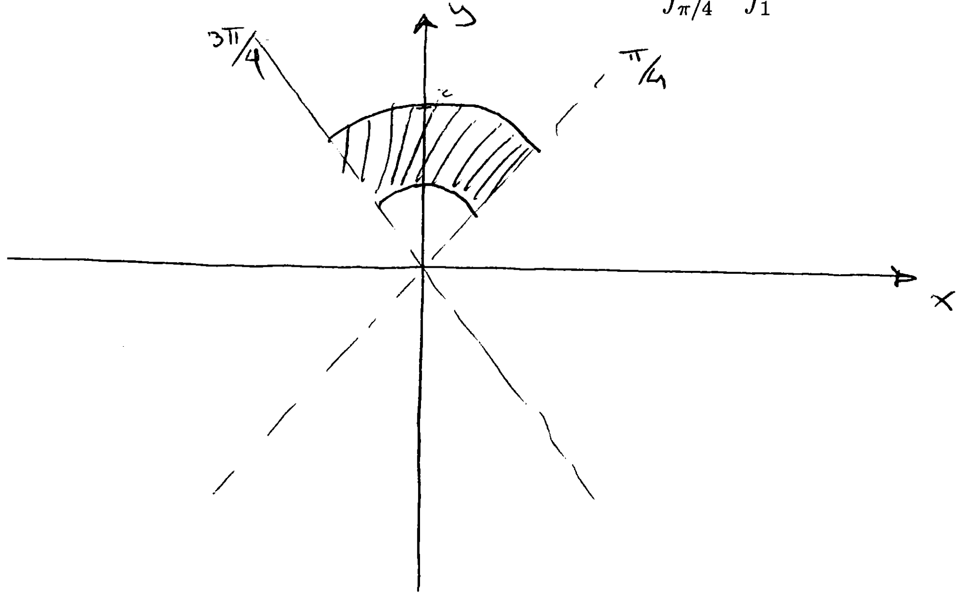
Solution: $\int_0^2 \int_y^{2y} xy\,dx\,dy = \int_0^2 \left[\frac{x^2 y}{2}\right]_y^{2y} dy = \frac{3}{2}\int_0^2 y^3\,dy = 6.$

3. (25 pts) Sketch the region of integration and change the order of integration of $\int_1^2 \int_0^{\ln x} f(x,y)\,dy\,dx$.



$$\int_1^2 \int_0^{\ln(x)} f(x,y)\,dy\,dx = \int_0^{\ln(2)} \int_{e^y}^{2} f(x,y)\,dx\,dy$$

4. (25 pts) Sketch the region whose area is given by the integral $\int_{\pi/4}^{3\pi/4} \int_1^2 r\,dr\,d\theta$.



Bonus (10 pts): Use iterated integrals in polar coordinates to find the volume of a sphere of radius a.

Let us denote by V the volume of the sphere of radius a. We will find the volume of the upper half hemisphere and multiply at the end by 2. Thus,

$\frac{V}{2} = \iint_D \sqrt{a^2 - x^2 - y^2}\,dA$, where D is the disk enclosed by $x^2 + y^2 = a^2$.

In polar coordinates:

$\frac{V}{2} = \int_0^{2\pi} \int_0^a \sqrt{a^2 - r^2}\, r\,dr\,d\theta$. Using $u = a^2 - r^2$, $du = -2r\,dr$ and therefore

$\frac{V}{2} = \frac{-1}{2} \int_0^{2\pi} \int_{a^2}^0 \sqrt{u}\,du\,d\theta = \frac{1}{2} \int_0^{2\pi} \int_0^{a^2} \sqrt{u}\,du\,d\theta$

$\frac{V}{2} = \frac{1}{2} \int_0^{2\pi} \frac{2}{3} u^{3/2} \Big|_0^{a^2}\, d\theta = \frac{1}{3} \int_0^{2\pi} a^3\,d\theta = \frac{a^3}{3}(2\pi) = \frac{2\pi a^3}{3}$.

Therefore, $V = \frac{4\pi a^3}{3}$.