University of Delaware
Department of Mathematical Sciences

MATH-529 – Fundamentals of Optimization
Instructor: Dr. Marco A. MONTES DE OCA
Spring 2013

Homework 1

Due date: February 13, 2012

Problems

1. Provide 10 examples of optimization problems that you tackle in your everyday life.
2. Write a 2-pages long paper about the gradient vector and directional derivatives. Cite your source(s).
3. Read the paper by Abdi, H. & Williams, L.J. (2010). Matrix algebra. In N.J. Salkind, D.M., Dougherty, & B. Frey (Eds.): Encyclopedia of Research Design. Thousand Oaks (CA): Sage. pp. 761-776. Available for download at `http://www.utdallas.edu/~herve/abdi-MatrixAlgebra2010-pretty.pdf`. Using this paper as main source, write a 2-page long (maximum) paper about eigendecomposition. If in addition to the aforementioned paper you use other sources, cite them accordingly.
4. Find the local and global minimizers and maximizers of a) $f(x) = x^2 + 2x$, b) $f(x) = x^4 + 4x^3 + 6x^2 + 4x$, and c) $f(x) = x + \sin x$.
5. Classify the following matrices according to whether they are positive or negative definite or semidefinite or indefinite:

a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ b) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

c) $\begin{pmatrix} 7 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ d) $\begin{pmatrix} 3 & 1 & 2 \\ 1 & 5 & 3 \\ 2 & 3 & 7 \end{pmatrix}$

e) $\begin{pmatrix} -4 & 0 & 1 \\ 0 & -3 & 2 \\ 1 & 2 & -5 \end{pmatrix}$ f) $\begin{pmatrix} 2 & -4 & 0 \\ -4 & 8 & 0 \\ 0 & 0 & -3 \end{pmatrix}$

6. Write the quadratic form $\boldsymbol{x} \cdot A\boldsymbol{x}$ associated with each of the following matrices A:

a) $A = \begin{pmatrix} -1 & 2 \\ 2 & 3 \end{pmatrix}$ b) $A = \begin{pmatrix} 2 & -3 \\ -3 & 0 \end{pmatrix}$

c) $A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & -2 & 2 \\ 0 & 2 & 3 \end{pmatrix}$ d) $A = \begin{pmatrix} -3 & 1 & 2 \\ 1 & 2 & -1 \\ 2 & -1 & 4 \end{pmatrix}$

7. Write each of the quadratic forms in the form $\boldsymbol{x} \cdot A\boldsymbol{x}$, where A is an appropriate symmetric matrix:

a) $9x_1^2 - 3x_2^2 + 2x_1x_2$
b) $x_1^2 - 3x_2^2 + 4x_3^2 - 4x_1x_2 + 6x_1x_3 + 2x_2x_3$
c) $3x_1^2 + x_2^2 + 2x_3^2 - x_1x_4 - x_1x_2 + 2x_1x_3$

8. Find the global minimizers and maximizers, if they exist, of the following functions:

a) $f(x_1, x_2) = x_1^2 - 4x_1 + 2x_2^2 + 7$
b) $f(x_1, x_2) = e^{-(x_1^2 + x_2^2)}$
c) $f(x_1, x_2) = x_1^2 - 2x_1x_2 + \frac{1}{3}x_2^3 - 4x_2$
d) $f(x_1, x_2, x_3) = (2x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - 1)^2$
e) $f(x_1, x_2) = x_1^4 + 16x_1x_2 + x_2^8$

9. Problem 2.3 of our textbook.

10. Suppose that A is a square matrix and suppose that there is another matrix B such that $A = B^T B$. Show that A is positive semidefinite.