University of Delaware
Department of Mathematical Sciences

MATH-529 – Fundamentals of Optimization
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Spring 2013

Homework 2

Due date: February 27, 2012

Problems

Note: You may use a computer algebra system to simplify your calculations. In all cases, however, you must explain your approach and conclusions.

1. Using the definition of convex function, check whether the following functions are concave, convex, strictly concave, strictly convex, or neither:

   a) $f(x) = x^2$ b) $f(\boldsymbol{x}) = x_1^2 + 2x_2^2$ c) $f(x, y) = 2x^2 - xy + y^2$

2. Using the fact that f has continuous first order derivatives, check whether the following functions are concave, convex, strictly concave, strictly convex, or neither:

   a) $f(x) = -x^2$ b) $f(\boldsymbol{x}) = (x_1 + x_2)^2$ c) $f(x, y) = -xy$

3. Using the fact that f has continuous first and second order derivatives, and that you may decompose a function into simpler functions (e.g., $f(x) = \alpha g(x) + h(x)$, $h(x) = g(f(x))$) check whether the following functions are concave, convex, strictly concave, strictly convex, or neither:

   a) $f(\boldsymbol{x}) = 2x_1^2 + x_2^2 + x_3^2 + 2x_2x_3$ b) $f(\boldsymbol{x}) = e^{x_1^2 + x_2^2 + x_3^2}$ c) $f(x, y) = 100(y - x^2)^2 + (1 - x)^2$

4. Determine the subset C of $\mathbb{R}^2$ over which the function $f(x, y) = x^2 - 4xy + 5y^2 - \ln xy$ is strictly convex.

5. Problem 2.8 of the textbook.