University of Delaware
Department of Mathematical Sciences

MATH-529 – Fundamentals of Optimization
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Homework 3

Due date: March 18, 2013

Problems

Note: You may use a computer algebra system to simplify your calculations. In all cases, however, you must explain your approach and conclusions.

1. Consider a quadratic function $f(\boldsymbol{x}) = a + \boldsymbol{b} \cdot \boldsymbol{x} + \frac{1}{2}\boldsymbol{x} \cdot A\boldsymbol{x}$, where $a \in \mathbb{R}$, $\boldsymbol{b} \in \mathbb{R}^n$, and A is a positive definite $n \times n$ matrix. Since $f(\boldsymbol{x})$ is strictly convex, it has a unique global minimizer $\boldsymbol{x}^\star$ that is the solution of the system $A\boldsymbol{x} = -\boldsymbol{b}$. Show that Newton's method with initial point $\boldsymbol{x}_0 \in \mathbb{R}$, reaches $\boldsymbol{x}^\star$ in one step.

2. Consider the function $f(\boldsymbol{x}) = \frac{1}{2}x_1^2 + \frac{a}{2}x_2^2$, with $a \geq 1$. Use the steepest descent method with exact directional minimization (that is, use α_k that minimizes $f(\boldsymbol{x}_k - \alpha_k \nabla f(\boldsymbol{x}_k))$ to verify the q-linearly convergent properties of the method in relation with the condition number of $\nabla^2 f(\boldsymbol{x})$. Your code should print the step length used at each iteration, $f(\boldsymbol{x}_k)$, $||\nabla f(\boldsymbol{x}_k)||$, and the approximation $\boldsymbol{x}_k$ at each iteration k. Run the program with $a = 2$ and $a = 20$. Discuss the results. Attach your code and the output of the program.

3. Problem 3.1 of the textbook using $\alpha_0 = 1$, $\rho = 0.4$, and $c = 0.0001$. In addition to printing the step length used by each method at each iteration, print also $f(\boldsymbol{x}_k)$, $||\nabla f(\boldsymbol{x}_k)||$, and the approximation $\boldsymbol{x}_k$ at each iteration k. Attach your code and the output of the program.

4. Problem 4.1 of the textbook.

5. Problem 4.2 of the textbook. Your program should output $\boldsymbol{x}_k$, $f(\boldsymbol{x}_k)$, $||\nabla f(\boldsymbol{x}_k)||$, and Δ_k. Comment your results.