University of Delaware
Department of Mathematical Sciences

MATH-529 – Fundamentals of Optimization
Instructor: Dr. Marco A. MONTES DE OCA
Spring 2013

Homework 4

Due date: April 10, 2013

Problems

1. Problem 12.16 of the book. (**I know the solutions manual is available online, so please refrain from using it. You will not learn if you let the author of the book solve your homework. Additionally, claiming others' work as your own is a serious academic honesty violation**.)
2. Problem 12.16 of the book using the Lagrange multipliers method, including the determinantal test for relative constrained extrema.
3. Problem 12.18 of the book. Substitute the text of point a) for "Use the Lagrange multipliers method to find all the critical points of this constrained problem."
4. Problem 12.20 of the book.
5. Use Lagrange multipliers to prove that the product of three positive numbers x, y, and z, whose sum has the constant value S, is a maximum when the three numbers are equal. Use this result to prove that $\sqrt[3]{xyz} \leq \frac{x+y+z}{3}$.
6. Suppose you want to design an ice-cube tray of minimal cost, which will make one dozen ice "cubes" (not necessarily actually cubes from 12 in.^{3} of water. Assume that the tray is divided into 12 square compartments in a 2×6 pattern as shown below, and that the material costs 1 cent per square inch. Use the Lagrange multipliers method to minimize the cost function $f(x, y, z) = xy + 3xz + 7yz$ subject to the constraints $x = 3y$ and $xyz = 12$. Apply the second-order test to verify that you do have a local minimum.