University of Delaware
Department of Mathematical Sciences

MATH-529 – Fundamentals of Optimization
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Spring 2013

Homework 5

Due date: April 29, 2013

Problems

1. Minimize x_1 subject to $x_1^2 - x_2 \geq 0$, x_1, $x_2 \geq 0$. Solve graphically and check whether the optimal solution satisfies the LICQ and MFCQ qualifications, as well as the KKT conditions.

2. Minimize $2x_1 + x_2$ subject to $x_1^2 - 4x_1 + x_2 \geq 0$, $-2x_1 - 3x_2 \geq -12$, x_1, $x_2 \geq 0$. Solve graphically for the global minimum and check whether that solution satisfies the LICQ and MFCQ qualifications, as well as the KKT conditions.

3. As you know, any trust region method has to solve the subproblem $\min_p m_k(p)$ subject to $||p|| \leq \Delta_k$. Suppose that the objective function being minimized is $f(x) = x_1^2 x_2^2 + x_1^2 - x_1$, that the current solution point is $x_k = (1, 0)^T$, and that $\Delta_k = 1/4$. Find all the points that satisfy the KKT conditions for this problem. Show that the Cauchy point is equal to the exact solution.

4. Repeat the previous exercise but assume now that $x_k = (1, 1)^T$. Remember that if the Hessian of the function at x_k is indefinite, you need to modify it so that the matrix used for the model $m_k(p)$ is positive definite. You may use software to solve the equations that will result from setting up the KKT conditions. Compare again the exact solution with the Cauchy point.

5. Some people say that time is our only asset. For those who agree, optimizing time allocation to accomplish certain tasks should be of paramount importance. With what you know about optimization, help Sandrine (the main character of the story below) optimize her time.

   Sandrine is a senior student at the University of Delaware. She works part time and is taking four courses. Final exams are approaching and she needs to get the best grades she can possibly can. A promotion might be at stake since her annual evaluation at work is in September and her GPA might be a differentiating factor with respect to other candidates. Unfortunately, she did not take MATH529 so she needs help from someone who did to solve her problem.

   Everyone knows that studying for an exam pays off, but up to a point. For example, the difference in grades obtained after studying for 1 hour vs. 2 hours is expected to be greater than the difference in grades obtained after studying for 10 hours vs. 11 hours. Also, certain courses are inherently harder than others, so no matter how hard one studies, the maximum expected grade will be most likely lower for a hard course than for an easy one. So the expected grade for a course is a function of the number of study hours and the difficulty of the course. If one wants to maximize the expected grade of Sandrine on her four courses, one also has to consider her time constraints. In fact, she can only devote a maximum 30 hours of study because she has to work.

Create a model that captures the nonlinearity of the study-time vs. grade phenomenon and the inherent difficulty of each course (the difficulty level of each course may be a given as a parameter of your model whose value can be estimated by polling students taking those courses). Using your model, find the optimal allocation of study time for the four courses in order to maximize Sandrine's grades without forgetting to take into account her time constraints.