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Section: 51

MATH 243 - Quiz 2

March 10, 2014

Please, SHOW ALL YOUR WORK as partial credit may be given; note all relevant equations, ideas, theorems, sketches, etc., to show what you know. Simplify wherever possible to make your answer more compact and neat. (Otherwise, if your answer cannot be simplified then leave it as is.) DO NOT leave your answer as a complex fraction. Answers without justification will be heavily penalized.

1. (25 pts) Are the lines with symmetric equations

$$x-1=y-2=3-z$$

and

$$-x=1-y=z-1$$

parallel, or not. Justify your answer.

Line 1 $\begin{cases} t=x-1, & t=y-2, & t=3-z \\ x=t+1, & y=t+2, & z=3-t \end{cases}$ : $\vec{r}_1(t)=\langle 1,2,3\rangle + t\langle 1,1,-1\rangle$

Line 2 $\begin{cases} t=-x, & t=1-y, & t=z-1 \\ x=-t, & y=1-t, & z=t+1 \end{cases}$ : $\vec{r}_2(t)=\langle 0,1,1\rangle + t\langle -1,-1,1\rangle$

Since $\langle 1,1,-1\rangle = -1\langle -1,-1,1\rangle$, the lines are parallel.

2. (25 pts) Where does the line $\langle 1,2,3\rangle + t\langle 1,1,-1\rangle$ intersect the plane $x+2y-z=0$?

$\vec{r}(t)=\langle 1+t, 2+t, 3-t\rangle \Rightarrow (1+t)+2(2+t)-(3-t)=0$

$1+t+4+2t-3+t=0$

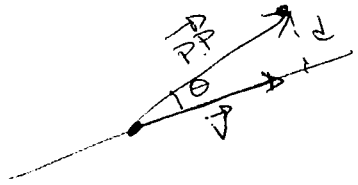
$4t+2=0 \Rightarrow t=-\frac{1}{2}$

Point:

$\left(1-\frac{1}{2}, 2-\frac{1}{2}, 3+\frac{1}{2}\right)$

$\left(\frac{1}{2}, \frac{3}{2}, \frac{7}{2}\right)$

3. (25 pts) What is the distance between the point $(1,1,1)$ and the line $\langle 1,0,0\rangle + t\langle 1,1,1\rangle$.




$d = \|\vec{PP}\|\sin\theta = \left|\dfrac{\|\vec{PP}\times\vec{v}\|}{\|\vec{v}\|}\right|$

$\vec{PP} = \langle 1-1, 1-0, 1-0\rangle = \langle 0,1,1\rangle$

$\vec{PP}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \langle 0,1,-1\rangle$

$\|\vec{PP}\times\vec{v}\| = \sqrt{2}$

$\|\vec{v}\| = \sqrt{3}$

$\therefore d = \dfrac{\sqrt{2}}{\sqrt{3}} = \sqrt{\dfrac{2}{3}}$

4. (25 pts) Find an equation of the line of intersection of the planes $x+2y-z=0$ and $2x-y-z=1$.

$\vec{n_1} = \langle 1,2,-1\rangle$
$\vec{n_2} = \langle 2,-1,-1\rangle$

$\Rightarrow \vec{n_1}\times\vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -1 & -1 \end{vmatrix} = \langle -2-1, -(-1+2), -1-4\rangle$

$= \langle -3,-1,-5\rangle$ ← direction vector

If $z=0$

$x+2y=0 \Rightarrow x=-2y$

$2x-y=1$

$2(-2y)-y=1$

$-4y-y=1 \Rightarrow y=-\dfrac{1}{5} \Rightarrow x=\dfrac{2}{5}$

$\Rightarrow$ point: $\left(\dfrac{2}{5}, -\dfrac{1}{5}, 0\right)$

Line:

$\vec{r}(t) = \left\langle \dfrac{2}{5}, \dfrac{-1}{5}, 0\right\rangle + t\langle -3,-1,-5\rangle = \left\langle \dfrac{2}{5}-3t, -\dfrac{1}{5}-t, -5t\right\rangle$