Name: Marco A. Montes de Oca
Section: 51

MATH 243 - Quiz 3
April 9, 2014

Please, SHOW ALL YOUR WORK as partial credit may be given; note all relevant equations, ideas, theorems, sketches, etc., to show what you know. Simplify wherever possible to make your answer more compact and neat. (Otherwise, if your answer cannot be simplified then leave it as is.) DO NOT leave your answer as a complex fraction. Answers without justification will be heavily penalized.

1. (25 pts) Find the equation of a plane tangent to the surface $z = xy + 3y$ at the point (1,2,8).

$f(x,y) = xy + 3y$ , $f_x(x,y) = y$ $f_y(x,y) = x + 3$

Tangent plane @ $(1,2,8)$

$z - 8 = f_x(1,2)(x-1) + f_y(1,2)(y-2)$

$z - 8 = 2(x-1) + 4(y-2)$

$z \cancel{-8} = 2x - 2 + 4y \cancel{-8}$

$z = 2x + 4y - 2$

2. (25 pts) Let $f(x,y) = \sin(xy) + x$, and $x = e^{u^2}$ and $y = \sin v$. Use the chain rule to find $\frac{\partial f}{\partial v}$.

$$\frac{\partial f}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v} = \left(\cos(xy)(y) + 1\right)(0) + \left(\cos(xy)(x)\right)\cos v$$

$$= x\cos(xy)\cos v = e^{u^2}\cos\left(e^{u^2}\sin v\right)\cos v$$

3. (25 pts) Let $f(x,y,z) = x^2 + y^2 + z^2 + xy + yz - 1$. What is the rate of change of $f$ at (1,0,2) in the direction of $\hat{u} = \langle \frac{4}{5}, \frac{3}{5}, 0 \rangle$?

$$D_{\hat{u}}f = \nabla f \cdot \hat{u} = \langle 2x+y, 2y+x+z, 2z+y \rangle \cdot \langle \tfrac{4}{5}, \tfrac{3}{5}, 0 \rangle$$

$$D_{\hat{u}}f(1,0,2) = \langle 2, 3, 4 \rangle \cdot \langle \tfrac{4}{5}, \tfrac{3}{5}, 0 \rangle = \frac{8}{5} + \frac{9}{5} = \frac{17}{5}$$

4. (25 pts) Find the points on the surface of the sphere $x^2 + y^2 + z^2 = 4$ such that a tangent plane to the sphere at those points is orthogonal to the line $\vec{r}(t) = \langle 1+8t, -2t, 1+4t \rangle$.

Let $f(x,y,z) = x^2 + y^2 + z^2 \Rightarrow \nabla f(x,y,z) = \langle 2x, 2y, 2z \rangle$

We want $\nabla f(x,y,z)$ to be parallel to $\langle 8, -2, 4 \rangle$ (the direction vector of the line. So

$$\langle 2x, 2y, 2z \rangle = \alpha \langle 8, -2, 4 \rangle \Rightarrow \begin{array}{l} 2x = \alpha 8 \Rightarrow x = 4\alpha \\ 2y = \alpha(-2) \Rightarrow y = -\alpha \\ 2z = \alpha 4 \Rightarrow z = 2\alpha \end{array}$$

Substituting $x, y, z$ in eq. of sphere:

$$(4\alpha)^2 + (-\alpha)^2 + (2\alpha)^2 = 4$$

$$16\alpha^2 + \alpha^2 + 4\alpha^2 = 4$$

$$21\alpha^2 = 4$$

$$\alpha^2 = \frac{4}{21}$$

$$\alpha = \pm\sqrt{\frac{4}{21}} = \pm \frac{2}{\sqrt{21}}$$

$\therefore$ Points:

$\left( \frac{8}{\sqrt{21}}, -\frac{2}{\sqrt{21}}, \frac{4}{\sqrt{21}} \right)$ and

$\left( -\frac{8}{\sqrt{21}}, \frac{2}{\sqrt{21}}, -\frac{4}{\sqrt{21}} \right)$