University of Delaware
Department of Mathematical Sciences

MATH-529 – Fundamentals of Optimization
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Spring 2014

Homework 1

Due date: February 24, 2014

Problems

1. Find the local and global minimizers and maximizers of $f(x) = x^4 - 2x^3 + x + 1$.
2. Find the local and global minimizers and maximizers of $f(x) = \sin\left(\frac{1}{x}\right)$.
3. Find the local and global minimizers and maximizers of $f(x) = 3x^5 - 10x^3 + 15x + 30$.
4. Find the local and global minimizers and maximizers of $f(\boldsymbol{x}) = x_1^3 - x_1^2 x_2 - x_1 x_2 + x_2^2$.
5. Find the local and global minimizers and maximizers of $f(\boldsymbol{x}) = x_1^2 + x_2^2 - 2x_1 x_2$.
6. Find the local and global minimizers and maximizers of $f(\boldsymbol{x}) = e^{x_1 - x_2} + e^{x_2 - x_1}$.
7. Find a_1, a_2, and a_3 such that $\begin{pmatrix} a_1 & a_2 & a_3 \\ 2 & 1 & 0 \\ 3 & 1 & 2 \end{pmatrix}$ is positive semidefinite.
8. Find a function $f(\boldsymbol{x})$ ($\boldsymbol{x} \in \mathbb{R}^3$) with no stationary points, and therefore with no global or local minimizers or maximizers.
9. Suppose that A is a square matrix and suppose that there is another matrix B such that $A = B^T B$. Show that A is positive semidefinite. Use this fact to write a MATLAB[1] function that returns a random $n \times n$ positive semidefinite matrix.
10. Using the code you wrote in the previous exercise, plot three randomly generated quadratic forms of two variables (again, using MATLAB). Attach a printout of your code with the resulting images. Do the surfaces look alike? Comment on your observations.

[1] A nice introduction to the basics of MATLAB programming can be found at `http://www.eng.ed.ac.uk/teaching/courses/matlab/`. You may use a free MATLAB-like package called Octave (`https://www.gnu.org/software/octave/`) to do this homework. If you have any questions, please ask me.