University of Delaware
Department of Mathematical Sciences

MATH-529 – Fundamentals of Optimization
Instructor: Dr. Marco A. Montes de Oca
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Homework 3

Due date: March 26, 2014

Problems

1. Are the following functions convex? (Justify your answer.) a) e^x in $(-\infty, \infty)$, b) $|x^3|$ in $(-\infty, \infty)$, c) $\ln x$ in $(0, \infty)$

2. Do some research and list all the operations (e.g., addition, subtraction. etc.) that preserve convexity. Add the necessary theory that justifies the inclusion of an operation in the list.

3. Using the fact a function is convex on a set C if $f(\lambda a + (1 - \lambda)b) \leq \lambda f(a) + (1 - \lambda)f(b)$ for all a, b in C and all $0 \leq \lambda \leq 1$, show that the function $f(x) = x^2 + x$ is convex.

4. Using any method you want, show that the function $f(\boldsymbol{x}) = 10e^{3x_1 - x_2} + 5e^{x_1^2 + x_2^2}$ is convex on $\mathbb{R}^2$.

5. Using the theory and code for linear least squares, find the parameters of a polynomial of 5th degree that best fits the data from the paper by A. Leike (2002). "Demonstration of the Exponential Decay Law Using Beer Froth," European Journal of Physics, Vol. 23, No. 1, pp. 21-26. The data is found at `http://www.stat.ufl.edu/~winner/datasets.html` under " Exponential Decay of Beer Foam for 3 Brands after pouring glass".

6. Using your favorite optimization algorithm, steepest descent or Newton's method, find the parameters of an exponential model to fit the data of the previous exercise.

7. Solve problem 12.16 of the book.

8. Find the extreme values of $f(x, y, z) = x + 2y$ subject to $x + y + z = 1$ and $y^2 + z^2 = 4$.

9. Find the extreme values of $f(x, y, z) = yz + xy$ subject to $xy = 1$ and $x^2 + z^2 = 1$.

10. Find the point on the line of intersection of the planes $x + y + z = 1$ and $2x - y - 3z = 3$ that is closest to the origin.