University of Delaware
Department of Mathematical Sciences

MATH-529 – Fundamentals of Optimization
Instructor: Dr. Marco A. Montes de Oca
Spring 2014

Homework 4

Due date: April 16, 2014

Problems

1. Research the significance of the bordered Hessian in mathematical economics models. Write a one page abstract about this topic.
2. Use a bordered Hessian to classify the solutions of the following problem:
$$\text{optimize } f(x, y) = xy \text{ subject to } x + 2y = 2.$$
3. Use a bordered Hessian to classify the solutions of the following problem:
$$\text{optimize } f(x, y) = xy \text{ subject to } x^2 + y^2 = 4.$$
4. Use a bordered Hessian to classify the solutions of the following problem:
$$\text{optimize } f(x, y) = x + y \text{ subject to } x^2 + y^2 = 4 \text{ and } x - y = 0.$$
5. Write down the KKT conditions that a solution to the following program must satisfy:
$$\text{minimize } f(x, y, z) = x^2 + y^2 - z^2 \text{ subject to } x + y - z \geq 2 \text{ and } x + z \geq 0.$$
6. Write down the KKT conditions that a solution to the following program must satisfy:
$$\text{minimize } f(\boldsymbol{x}) = 3x_1 + x_1^3 + 2x_2 + \frac{1}{3}x_2^3$$
subject to
$$-x_4 + x_3 - 1 \geq 0$$
$$-x_3 + x_4 - 1 \geq 0$$

$$100\sin(-x_3-1)+100\sin(-x_4-1)+50-x_1=0$$

$$100\sin(x_3-1)+100\sin(x_3-x_4-1)+50-x_2=0$$

$$x_1, x_2, x_3, x_4 \geq 0$$

7. Use the KKT conditions to find the solutions to the following program:

$$\text{minimize } f(x,y) = xy - x^2 - y^2$$

   subject to

$$x^2 + y^2 \leq 2$$

$$x + y \geq 0$$

8. Maximize $x_1 + x_2$ subject to $x^2 - y >= 0$, $-x^2 - y + 10 \geq 0$,$x \geq 0$,$y \geq 0$. Solve graphically and check whether the optimal solution satisfies the LICQ and MFCQ qualifications, as well as the KKT conditions.

9. Minimize x_1 subject to $x^2 - y >= 0$, $-x^2 - y + 10 \geq 0$,$x \geq 0$,$y \geq 0$. Solve graphically and check whether the optimal solution satisfies the LICQ and MFCQ qualifications, as well as the KKT conditions.

10. Minimize $2x_1 + x_2$ subject to $x_1^2 - 4x_1 + x_2 \geq 0$, $-2x_1 - 3x_2 \geq -12$, x_1, $x_2 \geq 0$. Solve graphically for the global minimum and check whether that solution satisfies the LICQ and MFCQ qualifications, as well as the KKT conditions.