University of Delaware
Department of Mathematical Sciences

MATH-529 – Fundamentals of Optimization
Instructor: Dr. Marco A. Montes de Oca
Spring 2014

Homework 5

Due date: May 14, 2014

Problems

1. Suppose you use a penalty method with quadratic penalty function and penalty parameter μ to tackle the problem:

$$\min \ -xy$$

subject to

$$x + 2y = 4$$

Determine the minimum value of μ to prevent the method to diverge (that is, find the minimum value of μ so that a minimizer exists). (Hint: Find the critical points of the penalized function in terms of μ and explore the implications of different values of μ.)

2. Suppose you use the augmented Lagrangian method to tackle the problem:

$$\min \ -\frac{1}{2}x^2$$

subject to

$$x = 1$$

Verify that the solution is $x = 1$ with $\lambda = 1$. Show that if $\mu < 1$, the augmented Lagrangian is unbounded below. Show also that for $\mu > 1$, the augmented Lagrangian function is strictly convex when considered as a function of x and has therefore a unique minimizer. What happens when $\mu = 1$?

3. Modify as necessary the augmented Lagrangian code on my website to solve

$$\min \ (1 - x)^2 + 100(y - x^2)^2$$

subject to

$$x^2 + y^2 = 1$$

Change the stopping criterion is such a way that the algorithm performs exactly 50 iterations. Report in tabular form the iteration number, the current estimate of the solution, the current value of the objective function, the current estimate of the Lagrange multiplier, the current value of the penalty parameter, and the current value of the constraint.

4. Solve using the Simplex Method (by hand):

$$\max p = 5x - 4y + 3z$$

subject to

$$5x + 5z \leq 100\,, 5y - 5z \leq 50\,, 5x - 5y \leq 50\,, x, y, z \geq 0.$$

5. Use MATLAB's optimization toolbox (linear programming commands) to solve this problem: You are in charge of purchases at the student-run used-book supply program at your college, and you must decide how many introductory calculus, history, and marketing texts should be purchased from students for resale. Due to budget limitations, you cannot purchase more than 650 of these textbooks each semester. There are also shelf-space limitations: Calculus texts occupy 2 units of shelf space each, history books 1 unit each, and marketing texts 3 units each, and you can spare at most 1,000 units of shelf space for the texts. If the used book program makes a profit of \$10 on each calculus text, \$4 on each history text, and \$8 on each marketing text, how many of each type of text should you purchase to maximize profit? What is the maximum profit the program can make in a semester?

6. Write down the dual problem associated with the following linear program:

$$\max p = x + y + z + w$$

subject to

$$x + y + z \leq 3\,, y + z + w \leq 4\,, x + z + w \leq 5\,, x + y + w \leq 6\,, x, y, z, w \geq 0.$$

7. Incumbent Tax N. Spend and challenger Trick L. Down are running for county executive, and polls show them to be in a dead heat. The election hinges on three cities: Littleville, Metropolis, and Urbantown. The candidates have decided to spend the last weeks before the election campaigning in those three cities; each day each candidate will decide in which city to spend the day. Pollsters have determined the following payoff matrix, where the payoff represents the number of votes gained or lost for each one-day campaign trip.

		T. N. Spend		
		Littleville	*Metropolis*	*Urbantown*
T. L. Down	**Littleville**	−200	−300	300
	Metropolis	−500	500	−100
	Urbantown	−500	0	0

What percentage of time should each candidate spend in each city in order to maximize votes gained? If both candidates use their optimal strategies, what is the expected vote?