MATH529 – Fundamentals of Optimization

Welcome!

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This Course

MATH

$$\alpha_k \mathbf{p}_k = -(Hf(\mathbf{x}_k))^{-1} \nabla f(\mathbf{x}_k)$$

CODE

```
    Wsize = 48;  sigma_d = 5;
    gd = fspecial('gaussian', [Wsize+1 Wsize+1], sigma_d);
    %another way of generating Guassian
    %[gdX, gdY] = meshgrid(-Wsize/2:Wsize/2, -Wsize/2:Wsize/2
    %gd = 1/(2*pi*sigma_d^2)*exp(-(gdX.^2+gdY.^2)/(2*sigma_d^
    %figure, surfc(gd);
    sigma_r = 40;
(0)
    for i=1:1:m1
        for j=1:1:n1
            %apply bilateral to R,G,B separately
            for k=1:1:d1
                %get the computation region, considering the
                Xst = max(1, j-Wsize/2);
                Xed = min(n1, j+Wsize/2);
                Yst = max(1, i-Wsize/2);
                Yed = min(m1, i+Wsize/2);
                IRegion = I1(Yst:Yed, Xst:Xed, k);
                %generate Gaussian filter based on intensity
                gr = 1/(2*pi*sigma_r^2)*exp((-(IRegion-I1(i,
                gr = gr./sum(gr(:));
                %figure, surfc(gr);
                %get the bilateral filter coefficients
                Bco = gd((Yst:Yed)+Wsize/2-i+1, (Xst:Xed)+Wsi
                %compute the value for filtered pixel
                I1base(i, j, k) = sum(sum(Bco.*IRegion))/sum(
            end
        end
    end
    %for debug: show the bilteral filtered image
```

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- **Every time we make a decision, we optimize!**

Optimization and Mathematics

We use functions to help us distiguish between the elements of the set of available alternatives.

Alternatives to choose from

$$\{x, y, z\}$$

$$\downarrow$$

$$\{f(x), f(y), f(z)\}$$

Numerical values indicating (un)desirability

f is usually called *objective function*; however, other names, such as cost function, loss function, fitness function, etc. may be used.

Optimization and Mathematics

Example:

$f(x_1) = 5.1$

$f(x_2) = 6$

$f(x_3) = 2.2$

Basic types of optimization problems

There are many different types of optimization problems depending on the features of the objective function f and/or the set of available alternatives.

The basic types of optimization problems are:

- If the set of available alternatives is finite, then the problem of finding the best element of that set is a **discrete optimization problem**.

Example

The problem of finding the best route from an origin to a destination is a discrete optimization problem.

Basic types of optimization problems

- If x is drawn from an uncountably infinite set (e.g., $\mathbb{R}$), then the problem is a **continuous optimization problem**.

Example

The problem of finding the price of a product that maximizes profit is a continuous optimization problem.

Basic types of optimization problems

- If the set of values that x can take are restricted, then the problem is a **constrained optimization problem**.

Example

When trying to find the best allocation of investments in different assets (portfolio optimization), we need to ensure that the sum of these investments is equal to the total available capital.

Note that constrained optimization problems may be continuous or discrete.

Basic types of optimization problems

- If measurements of $f(x)$ are affected by noise, then the problem is a **stochastic optimization problem**.

Example

In the portfolio optimization problem, the return rate associated with each asset is a random variable, so the same allocation would give different returns at different times. These kinds of problems are stochastic optimization problems.

Note that stochastic optimization problems may be constrained and continuous, or discrete.

Basics

Problem Statement

An optimization problem can be stated as follows: Given are a certain set X and a function f which assigns to every element of X a real number. The optimization problem consists in finding an element $x^\star \in X$ such that

$$f(x^\star) \leq f(x) \text{ for all } x \in X\,.$$

The set X is referred to as the *feasible set*, and the function f is called the *objective function*.

Typically, X will be a subset of the space $\mathbb{R}^n$ and f will be relatively regular (e.g., differentiable). The definition of X will be based on systems of equations and inequalities called *constraints*.

Notation

The standard notation used to represent an optimization problem is:

$$\min_{\mathbf{x}} \; f(\mathbf{x})$$

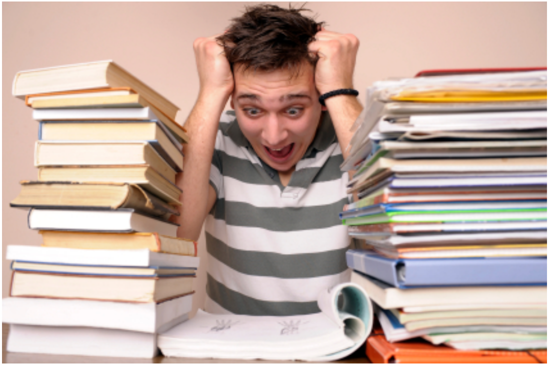
subject to

$$g_i(\mathbf{x}) \geq 0, \;\; i \in \mathcal{I} = \{1, \ldots, m\}$$
$$h_j(\mathbf{x}) = 0, \;\; j \in \mathcal{E} = \{1, \ldots, p\}$$

where

$$\mathbf{x} = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$$

Example: Studying for Finals



Meetings & Contact

Instructor:	Dr. Marco A. Montes de Oca
Office:	315 Ewing Hall
Phone:	302-831-7431
Email:	mmontes@math.udel.edu
URL:	`http://math.udel.edu/~mmontes/teaching/UD/S14-MATH529-10.html` `https://sakai.udel.edu/portal/`
Office hours:	Mondays 5:00pm–7:00pm or by appointment
Meetings:	Mondays and Wednesdays 3:35pm–4:50pm, 330 Purnell Hall

Evaluation

The final grade components are: Homeworks, Exams, and a Project.

The contribution of each component is as follows:

Component	Weight
Homeworks	30%
Exam 1	15 %
Exam 2	15 %
Final Exam	20 %
Project	20 %