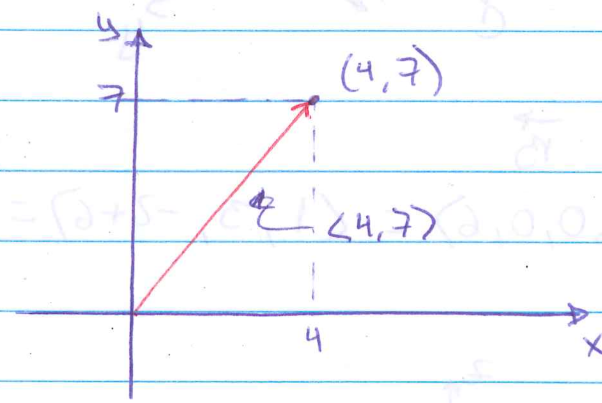
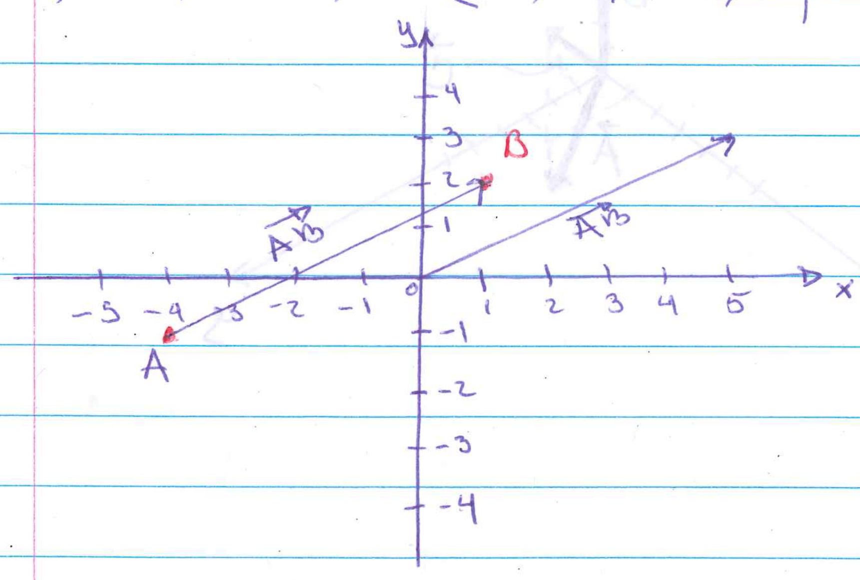
# Homework #2

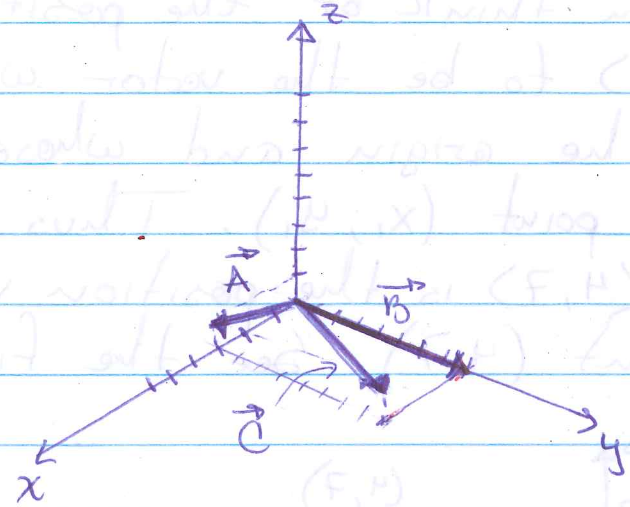
1. You can think of the position vector $\langle x_1, y_1 \rangle$ to be the vector whose tail is at the origin and whose head is at the point $(x_1, y_1)$. Thus, the vector $\langle 4,7 \rangle$ is the position vector of the point $(4,7)$. See the figure below.



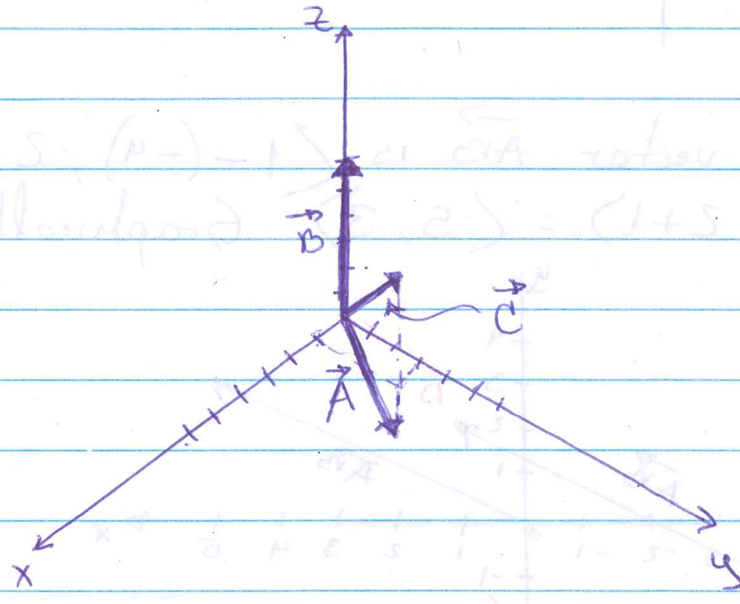

2. The vector $\vec{AB}$ is $\langle 1-(-4), 2-(-1) \rangle = \langle 1+4, 2+1 \rangle = \langle 5, 3 \rangle$. Graphically



3. $\underbrace{\langle 3,0,1\rangle}_{\vec{A}} + \underbrace{\langle 0,8,0\rangle}_{\vec{B}} = \underbrace{\langle 3,8,1\rangle}_{\vec{C}}$




4. $\underbrace{\langle 1,3,-2\rangle}_{\vec{A}} + \underbrace{\langle 0,0,6\rangle}_{\vec{B}} = \langle 1,3,-2+6\rangle = \underbrace{\langle 1,3,4\rangle}_{\vec{C}}$



5. $\vec{a} = 4\hat{i} + \hat{j}$ , $\vec{b} = \hat{i} - 2\hat{j}$

a) $\vec{a} + \vec{b} = (4\hat{i} + \hat{j}) + (\hat{i} - 2\hat{j}) = (4+1)\hat{i} + (1-2)\hat{j}$
$= \boxed{5\hat{i} - \hat{j}}$

b) $2\vec{a} + 3\vec{b} = 2(4\hat{i} + \hat{j}) + 3(\hat{i} - 2\hat{j}) = 8\hat{i} + 2\hat{j} + 3\hat{i} - 6\hat{j} =$
$= \boxed{11\hat{i} - 4\hat{j}}$

c) $|\vec{a}| = \sqrt{(4)^2 + (1)^2} = \sqrt{16+1} = \boxed{\sqrt{17}}$

d) $|\vec{a} - \vec{b}| = |(4\hat{i} + \hat{j}) - (\hat{i} - 2\hat{j})| = |3\hat{i} + 3\hat{j}| = \sqrt{3^2 + 3^2} =$
$= \sqrt{9+9} = \sqrt{18}$
$= \boxed{3\sqrt{2}}$

6. $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ , $\vec{b} = -2\hat{i} - \hat{j} + 5\hat{k}$

a) $\vec{a} + \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + (-2\hat{i} - \hat{j} + 5\hat{k}) =$
$(1-2)\hat{i} + (2-1)\hat{j} + (-3+5)\hat{k} =$
$\boxed{-\hat{i} + \hat{j} + 2\hat{k}}$

b) $2\vec{a} + 3\vec{b} = 2(\hat{i} + 2\hat{j} - 3\hat{k}) + 3(-2\hat{i} - \hat{j} + 5\hat{k}) =$
$(2\hat{i} + 4\hat{j} - 6\hat{k}) + (-6\hat{i} - 3\hat{j} + 15\hat{k}) =$
$(2-6)\hat{i} + (4-3)\hat{j} + (-6+15)\hat{k} =$
$\boxed{-4\hat{i} + \hat{j} + 9\hat{k}}$

c) $|\vec{a}| = \sqrt{(1)^2 + (2)^2 + (-3)^2} = \sqrt{1+4+9} = \boxed{\sqrt{14}}$

d) $|\vec{a} - \vec{b}| = |(\hat{i} + 2\hat{j} - 3\hat{k}) - (-2\hat{i} - \hat{j} + 5\hat{k})| =$

$|(1+2)\hat{i} + (2+1)\hat{j} + (-3-5)\hat{k}| =$

$|3\hat{i} + 3\hat{j} - 8\hat{k}| = \sqrt{3^2 + 3^2 + (-8)^2} =$

$= \sqrt{9+9+64} = \boxed{\sqrt{82}}$

7. Let $\vec{a} = -3\hat{i} + 7\hat{j}$. $|\vec{a}| = \sqrt{(-3)^2 + 7^2} = \sqrt{9+49} =$

$= \sqrt{58}$

A vector that has the same direction as $\vec{a}$ and has length equal to one can be found by dividing $\vec{a}$ by $|\vec{a}|$. Such a vector is

$$\boxed{\frac{1}{\sqrt{58}}(-3\hat{i} + 7\hat{j})}$$

8. Let $\vec{a} = 8\hat{i} - \hat{j} + 4\hat{k}$. Let $\vec{v} = \frac{\vec{a}}{|\vec{a}|} = \frac{8\hat{i} - \hat{j} + 4\hat{k}}{\sqrt{8^2 + (-1)^2 + 4^2}} =$

$\frac{8\hat{i} - \hat{j} + 4\hat{k}}{\sqrt{64+1+16}} =$

$\frac{1}{\sqrt{81}}(8\hat{i} - \hat{j} + 4\hat{k}) =$

$$\boxed{\frac{1}{9}(8\hat{i} - \hat{j} + 4\hat{k})}$$

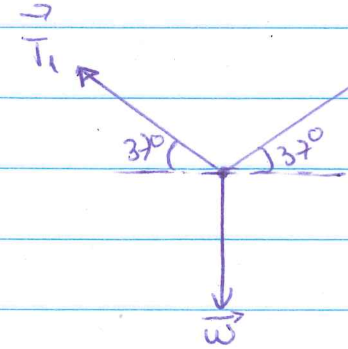
9. Let $\vec{a} = \langle -2, 4, 2 \rangle$. A unit vector with the same direction as $\vec{a}$ is

$$\vec{v} = \frac{\vec{a}}{|\vec{a}|} = \frac{\langle -2,4,2 \rangle}{\sqrt{(-2)^2+4^2+2^2}} = \frac{\langle -2,4,2 \rangle}{\sqrt{4+16+4}} = \frac{1}{\sqrt{24}} \langle -2,4,2 \rangle$$

The vector $\vec{v}$ has length equal to one. To have a vector with the same direction as vector $\vec{a}$ but with length six, we multiply $\vec{v}$ by 6.

$$\boxed{6\vec{v} = \frac{6}{\sqrt{24}} \langle -2,4,2 \rangle}$$

10. Let's assume that the weight of the chain is concentrated in a single point. Then the forces acting upon the chain can be represented as follows:




Since $|\vec{T_1}| = |\vec{T_2}| = 25$ N

$\vec{T_1} = -25\cos(37°)\hat{i} + 25\sin(37°)\hat{j}$

$\vec{T_2} = 25\cos(37°)\hat{i} + 25\sin(37°)\hat{j}$

Since the system is in equilibrium, we have

$$\vec{T}_1 + \vec{T}_2 = -\vec{w}$$

Therefore

$$\left(-25\cos(37^\circ)\,\hat{i} + 25\sin(37^\circ)\,\hat{j}\right) + \left(25\cos(37^\circ)\,\hat{i} + 25\sin(37^\circ)\,\hat{j}\right)$$

$$= -|w|\,\hat{j}$$

$$\therefore\ |w| = 50\sin(37^\circ) = \underline{30.1\ N}$$